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DESIGN CRITERIA FOR THE PREDICTION AND PREVENTION OF PANEL FLUTTER

VOLUME II: BACKGROUND AND REVIEW OF STATE OF THE ART

CLARK E. LEMLEY

McDonnell Douglas Corporation

TECHNICAL REPORT AFFDL-TR-67-140

AUGUST 1968

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The distribution of this report is limited because it contains the development of improved design criteria for the prevention of panel flutter applicable to future aerospace systems.

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VOLUME II: BACKGROUND AND REVIEW OF STATE OF THE ART

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FOREWORD

This report, prepared by the Structural Dynamics Department of the Engineering Technology Division of McDonnell Douglas Corporation, covers work performed under Air Force Contract AF33(615)-5295. The contract was sponsored by the Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. This work was done to provide improved panel flutter design criteria for aircraft and aerospace vehicles as part of the exploratory research program of the Air Force Systems Command. This research was conducted under Project No. 1370, "Dynamic Problems in Flight Vehicles," and Task No. 137003, "Prediction and Prevention of Aerothermoelastic Problems." This report covers work conducted from August 1966 to November 1967. The work was administered by Mr. Michael H. Shirk of the Vehicle Dynamics Division.

The program was managed by Dr. Norman Zimmerman. Dr. Clark E. Lemley was the principal investigator. Significant technical contributions to the program were made by Mr. Bobby R. Scheller, Structural Dynamics Engineer.

The manuscript was released by the author in January 1968 for publication.

This technical report has been reviewed and is approved.

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ABSTRACT

The program described in this report was performed to bring together all available data from wind tunnel test, flight test, vibration test, thermal test and theoretical investigations to form comprehensive panel flutter design criteria. Procedures were developed which are applicable to the environment and various panel structural arrangements for transonic and hypersonic aircraft, aerospace re-entry vehicles, and boosters.

This report (Volume II) presents the results of investigations to determine the state of the art in panel design and to provide the background data for the criteria that are given in Volume I. The investigations included a thorough literature search and review as well as surveys of personnel and facilities having made recent contributions in the field. In addition, supplementary analyses are described that were required in some areas to complete the criteria presentation. A comprehensive bibliography is appended to this volume.

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LIST OF SYMBOLS

A_n, Γ_m	Generalized Coordinates
a_{∞}	Speed of sound
[illegible]	Aerodynamic integral function $= \int_0^{\ell} (\partial F_m/\partial x) F_r dx$
C	Constant
C_{rm}	Constant proportional to elastic coupling between modes r and m
C_{rr}	Modal frequency constant $= \dfrac{\rho_p t \ell^4 \omega^2_{rr}}{D}$
D	Plate bending stiffness $= \dfrac{Et^3}{12(1-\nu^2)}$
d	Cavity depth
d_o	Static defelction
E	Modulus of elasticity
e	Natural logarithm
f( )	Function of
F_m	Beam vibration mode shape function used to describe panel displacement along the x-axis
f_m	Still air frequency of a panel mode with m streamwise half waves
f(M)	Mach number correction factor
G_{rs}	Constant for Curved Panel (Table I, Page 32)
G_n	Beam vibration mode shape function used to describe panel displacement along the y-axis
i	$\sqrt{-1}$
h_o	Crown height of curved panel
k_{rm}	Elastic derivative, generalized stiffness
k_{11}	Vibration frequency parameter (From Reference 16)
ℓ	Panel length (streamwise)
ℓ'	Streamwise length of a panel yawed 90^o
M	Mach number

m	Streamwise mode number
N	Curvature parameter ($= \frac{h_o}{t}$)
N_x	Inplane load (streamwise)
N_y	Inplane load (cross-stream direction)
n	Cross-stream mode number
p	Aerodynamic pressure
p_α	Static aerodynamic pressure due to angle of attack
Δ_p	Differential pressure
Q_n	Generalized force
q	Dynamic pressure ($= \frac{1}{2}\rho V^2$)
R	Radius of curvature
ΔT	Differential temperature
t	Panel thickness
t	Time
T	Kinetic energy
U_{int}	Internal energy
U_M	Stretching energy
U	Potential energy
U_B	Bending energy
V	Velocity
W	Work
W_{ext}	External work
w	Panel width
w	Displacement of a point in the panel measured normal to the panel plane
w'	Cross-stream width of a panel yawed 90°
w_s	Static panel deflection due to differential pressure

x	Direction parallel to air flow when $\Lambda = 0$
y	Direction perpendicular to air flow when $\Lambda = 0$
α	Angle of attack
α_T	Coefficient of thermal expansion
β	Compressibility parameter $\left(= \sqrt{M^2 - 1}\right)$
λ	Nondimensional dynamic pressure parameter ($= \frac{2q\ell^3}{\beta D}$)
δ	Variational operator
δ_{rm}	Kronecker delta function (= 1 if r = m; = 0 if r $\neq$ m)
ϵ	Axial strain
θ	Sector angle of cylindrically curved panel
Λ	Yaw or sweep angle
γ	Eigenvalue of the characteristic equation defined by the flutter determinant ($= [\rho t\ell^4/D]\omega^2$)
λ_c	Nondimensional cavity parameter (from Reference 16)
ρ	Mass density of air
ρ_p	Mass density of panel
σ	Inplane stress
τ_{xy}	Shearing stress
ν	Poisson's ratio
ϕ	Nondimensional panel flutter parameter ($= \left[\frac{f(M)E}{q}\right]^{1/3} \frac{t}{\ell}$)
ω	Frequency (radians)
ω_o	Reference frequency, two-dimensional simply supported panel

SUBSCRIPTS

A	Aerodynamic
B	Baseline
cav	Cavity
cr	Critical

eff	Effective
L	Local
m	Streamwise mode number
max	Maximum
min	Minimum
n	Cross-stream mode number
N_x	Inplane load (streamwise)
$N_{x_{buckling}}$	Buckling load (streamwise)
Δ	Increment of change in a parameter
vac	Vacuum
∞	Free stream

SECTION I

INTRODUCTION

The purpose of the work described in this report was to update existing criteria and, where possible, to develop new criteria for designing panels that may be subject to flutter. The criteria and guidelines presented in this document are based on (a) the results of literature surveys, (b) consultation with personnel who have made recent contributions in the field, and (c) supplementary analyses that were needed to clarify trends. The panel parameters and physical characteristics that are taken into account are the following:

Mach number	Edge conditions
Dynamic pressure	Curvature
Angle of attack	Modulus
Length	Inplane stress
Width	Differential pressure
Thickness	Differential temperature
Sweep (yaw) angle	Cavity effect

The work is divided into two volumes in order to facilitate use of the criteria; Volume I presents the criteria and guidelines for panel design, and Volume II contains the background data and study results that were used as bases for the criteria as well as discussions of inherent shortcomings and problem areas.

In achieving the objectives of the program, the effort was divided into the major phases as follows:

1. Data Compilation and Evaluation

This phase consisted of: (a) a comprehensive literature search and survey to define the state of the art as pertaining to published information; (b) contacts (personal and/or telephone) with individuals and facilities that have been active in panel flutter work whether in research, test, or design; and (c) evaluation and correlation of the data and information obtained during the literature and facilities surveys.

2. Parametric and Trend Studies

The specific objectives of this phase included: (a) isolation of the trends obtained during the first phase data collection; (b) the supplementary analyses that were necessary to complete the criteria for design of flutter-free panels; (c) definition of comprehensive design criteria; and (d) definition of areas, both analytical and experimental, in which further work would improve the state of the art.

3. Formulation of Panel Design Techniques

This phase included: (a) study of existing design techniques and existing methods of presentation; (b) formulating a set of comprehensive design criteria that incorporate elements of existing criteria together with results of supplemental studies; and (c) definition of areas that require further study.

The facilities survey is discussed in Appendix A to this volume and includes discussions of both the information obtained and of the panel flutter design criteria that are in current use.

The literature survey resulted in the extensive bibliography that is presented in Appendix C. In addition, certain highlights and milestones from this survey have been incorporated into the historical sketch that is presented in the following paragraphs.

4. Historical Developments

The first recognition of panel flutter is credited to Jordan (1)* who attributed the flight failures of some sixty to seventy V-2 rockets to an aeroelastic instability of the outer skins. The first analysis of panel flutter appears to be the interesting study of Isaacs (2) in which he investigated the behavior of a buckled panel exposed to supersonic flow. The first buckling mode is a statically stable configuration in still air, but Isaacs showed that static aerodynamic forces cause the first mode to take on the appearance of and coalesce with the second buckling mode. Since the second mode is not statically stable, he reasoned that panel flutter must occur when static stability is no longer possible. Miles (3) presented the first study of panel flutter as a dynamic phenomenon in 1950. He used linearized quasi-steady aerodynamic theory together with an assumed mode representation of a two-dimensional panel. He showed that aerodynamic damping was negative between $M = 1.0$ and $M = \sqrt{2}$ and predicted that all panels would flutter in that range regardless of thickness. Shen (4) (5) extended Miles' work on the two-dimensional simply supported panel by using an exact, rather than an assumed mode, approach and found that increasing the panel thickness had a stabilizing effect even in the transonic range. In addition, Shen made improvements on the generalized aerodynamic forces for sine mode shapes so that better agreement could be obtained with the exact theory. A test program of significant scope was undertaken by the National Advisory Committee for Aeronautics and the Langley Aeronautical Laboratory when wind tunnel tests reported by Sylvester and Baker (6) were conducted to verify the existence of panel flutter and to study the effects of some structural parameters on the aeroelastic characteristics. Two panel configurations were tested; the first was clamped front and rear but free on the edges so that tension could be applied, and the second was clamped on all four edges thereby simulating a more realistic structural configuration. Both types of panels fluttered. On the basis of experimental evidence from the Langley tests it was concluded that flutter could be eliminated by applying sufficient tensile stress, by decreasing streamwise length, by increasing bending stiffness, or by applying a differential pressure (that is, static overpressure applied to one side) across the panel to build up tensile stress.

In 1954 Goland and Luke (7) used the Laplace transformation to analyze the stability of a two-dimensional membrane. By using potential theory aerodynamics they showed that a membrane will not flutter at high Mach numbers, contrary to the prediction of the Galerkin method. Because of the spurious

* Numbers in parentheses indicate References listed at the end of this volume.

boundaries that the Galerkin procedure predicted for membranes, they reasoned that the same result would hold true for panels and suggested that all panels might become stable at the high Mach numbers. They did not actually extend their analysis to panels; however, Hedgepeth, Budiansky, and Leonard (8) studied a multibay panel configuration for which the supports were assumed to be equally spaced along the direction of airflow. A noteworthy result of the study was the prediction that viscous damping is not always stabilizing in panel flutter but sometimes results in modal phase shifts that cause the panel to absorb more energy from the airstream than is dissipated by the damping.

The doubt cast on the validity of the Galerkin procedure came under attack by several investigators in early 1957. A part of the impetus may have been provided by the confidence that was gained in the point-function aerodynamic theories when Ashley and Zartarian (9) published their article on piston theory. They not only justified its use for certain combinations of Mach number, reduced frequency and local slope, but also showed how several problems in aeroelastic stability and response might be formulated. Hedgepeth (10) used quasi-static aerodynamic theory to obtain both closed form and assumed mode solutions for a simply supported plate of finite aspect ratio. His closed form solution showed that panels do flutter at high Mach numbers even though membranes do not. Furthermore he showed that boundaries obtained from two, three and four mode Galerkin analyses converged to the exact solution. It is to be noted that the eigenvalues that Hedgepeth obtained with the closed form solution showed the classical frequency coalescence as airspeed became critical. Hedgepeth's work was among the first to restore confidence in the Galerkin procedure, but many investigators have continued to use more elegant solutions thereby sacrificing a great deal of insight and flexibility that is only provided by a modal approach. From a practical standpoint, the advantage in using assumed modes lies in the ability to vary modal frequencies in the flutter study. The frequencies may be measured in still air tests, calculated, or varied in a parametric manner to find the effect on flutter boundaries. Cunningham (11), in investigating possible cavity effects on flutter, showed a strong destabilizing trend when the still air frequency of the fundamental mode was increased and allowed to coincide with the still air frequency of the second panel mode.

Although a clearer picture of the panel flutter phenomenon was emerging from the analyses, there was a real need for more experimental data to verify the accuracy of the theories. Unfortunately, the Mach 1.3 data of Sylvester and Baker did not agree well with any of the theories. In order to extend the experimental investigation of buckled panels, additional testing was performed at Langley in 1955 and the range of Mach numbers was extended to include 1.2 to 3.0. Sylvester (12) reported that the data for the buckled panels showed scatter due to variation in the type and amount of buckle. The experimental value of panel thickness to prevent flutter was some twenty percent lower than the value predicted by Isaacs' transtability theory, (2). One reason for the difficulties in correlating theory and experiment was the lack of coordination between the theoreticians and the experimentalists in formulating test programs. It was at this time that Fung and his associates at the California Institute of Technology began an extensive program of investigation

to incorporate both analysis and test. Included in the program were studies of a two-dimensional buckled plate (13), transonic flutter of a clamped-free panel (14), and supersonic tests of flat and curved panels (15). By the use of carefully controlled tests, the overall program contributed greatly to the understanding of the mechanisms of panel flutter and showed improvement in the correlation between theory and experiment.

More recent work has been devoted to improving test techniques (16) and (17), and to defining the areas of strength and weakness in analysis. Bohon and Dixon (18) have shown that two-dimensional static aerodynamic theory is applicable to all unstressed panels with length-to-width ratios greater than one. In addition, they indicate that the inclusion of structural damping in the analysis of stressed panels improves the comparison with experiment. Furthermore, these investigators point out that peculiarities in edge attachments of built-up, corrugated panels can lead to significant decreases in flutter boundaries. It is generally agreed that structural analyses of actual panels is the area that still requires the most work, and the trend of the recent investigations is strongly slanted toward that end (19)-(21).

SECTION II

EVALUATION OF CURRENT DESIGN TECHNIQUES

This Section presents an evaluation of the existing techniques that have been used to achieve the current state of the art in panel flutter. It is divided into three parts, and treats analyses, tests, and existing criteria.

1. Evaluation of Analytical Methods

There are two practical questions to which a designer would like reliable and predictable answers. The first concerns the actual location of the panel flutter boundary, i.e., the flight speed that separates stable from unstable behavior. The second concerns the behavior of the panel within the unstable flight regime, i.e., the nature of the instability as regards destructiveness, violence, annoyance, etc.

It is the first of these questions, concerning flutter boundary location, that is the major goal of the analytical efforts to be discussed here. The boundary for panel flutter is predicted by linear analysis because all large amplitude flutter begins first as infinitesimal motion and grows with time.

The post-flutter behavior, on the other hand, involves restoring stresses that are not linear with the amplitude of motion and, therefore, require analytical methods that are beyond the scope of this report.

a. General

The current methods for panel flutter analysis can be broadly grouped into two categories designated simply (a) exact and (b) approximate. The exact methods, usually more tedious to apply, are obtained by solving differential (or integro-differential) equations of motion and no recourse need be made to assumptions of mode shapes or frequencies. Discussions of the exact methods are presented in Reference, [11] and [22], and extensive data from these methods are presented in References [10] and [23]. The approximate methods that are in general use are usually referred to as "assumed mode" analyses and are applied to both the Galerkin and Rayleigh-Ritz solutions. In the latter cases the analyst uses panel modes with appropriate boundary conditions as allowed degrees-of-freedom and obtains approximate aeroelastic solutions to either the differential equation (Galerkin) or the system energy equations (Rayleigh-Ritz).

The reader may reasonably ask why approximate solutions should be required when exact methods are available. Each of these methods of analysis has its particular advantages. The exact methods, though more tedious to apply, provide trend data for certain types of panels (notably very high ℓ/w) that are obtained from the approximate methods with extreme difficulty. A serious shortcoming, however, is that measured mode shapes and frequencies cannot be used in the exact analysis and it is extremely difficult to account for anomalies in the panel behavior. In summary, the exact method is best adapted to treating ideal panels.

The approximate (assumed mode) methods are very useful in the analysis of panel configurations that can be described by including a reasonable number of assumed modes. (It is shown in Reference 24 that panels of very large ℓ/w require an unwieldy number of modes before satisfactory flutter solutions are obtained.) Anomalous panel behavior, as measured in vibration tests for example, can easily be incorporated in flutter analysis. Furthermore, the approximate methods are amenable to parametric trend studies from which the analyst is able to determine the effect of certain parameters on stability boundaries. In this report, data from both exact and approximate methods are used as aids in establishing criteria.

It would not be prudent to leave a discussion of analytical methods in panel flutter without mentioning the "membrane dilemma" (Reference 9) which casts doubt on the approximate methods. Briefly, it can be shown by exact analysis that a membrane (lacking bending rigidity) will not flutter; however, any finite number of modes used in an approximate analysis yields finite flutter boundaries. The addition of higher frequency modes always raises the predicted boundary. Studies of the behavior of panels of large ℓ/w indicate that the "membrane dilemma" is not caused by a lack of bending stiffness, but is due rather to the spacing of the frequencies between adjacent, coupled modes. A similar problem arises in any assumed mode panel analysis in which the frequency spectrum becomes altered in such manner to cause a uniform spacing of frequencies. Assumed mode analyses for $\ell/w > 4$ are of little value because convergence to an "exact" solution requires an inordinately large number of modes. The spacing of streamwise panel frequencies is governed by

$$f_m = C \left[m^2 + (\ell/w)^2\right]$$

in which f_m is frequency, C is a constant, m is streamwise mode number and ℓ/w is length-to-width ratio. It can be seen that the frequencies become closely spaced when $\ell/w \gg m$.

b. Aerodynamics

Panel flutter is a result of the interaction between aerodynamic and structural forces. It is generally believed throughout the industry that the aerodynamic forces are adequately predicted by theory and that most of the deficiencies in panel flutter prediction are due to inadequacies in analyzing the structure. This belief is supported by the fact that better theoretical correlation is obtained in tests of very simple panels than with structurally complicated (built-up, corrugated) panels. An excellent discussion and comparison of the aerodynamic theories used in panel flutter analysis is presented in Reference 18. It is pointed out, therein, that for $M > 1.3$ and $1 < \ell/w < 10$, two dimensional "strip" theory gives results that are in good agreement with three-dimensional "surface" theory aerodynamics. For $M < 1.3$, the three-dimensional theory (based on linearized potential flow theory) must be used to obtain results that correlate with test data. Such an analysis is described in Reference 25. For $M > 2$ the point function aerodynamic theories (such as the well known "piston" theory, or Ackeret theory) offer reasonable approximations to the local aerodynamic pressure and greatly simplify the analyses.

c. Structures

The principal shortcomings of most of the structural analyses are apparent when predicted still air dynamics are compared with bench test vibration data. The deficiencies in the theory are due to mathematical idealizations which fail to describe the real panel characteristics. "Anomalous" behavior of the real panels can often be traced to conditions such as:

Deviation from flatness

Inhomogeneities (mass, stiffness)

Edge conditions

Studies recently conducted at NASA Langley by Guy and his associates have shown much promise in isolating the effects of edge supports on the vibration and flutter of built-up panels. The designer should keep in mind that the "stiffness" or "softness" of edge supports only has meaning in relation to the elasticity of the panel itself.

Difficulties that are encountered in the analytical description of the dynamics of plate-like structures are due largely to (a) unanticipated inplane stresses and (b) edge conditions that are inadequately described. It is virtually impossible for the analyst to assess the accuracy of his analysis without test data. He has the possibility of evaluating the accuracy of his effort with measured vibration data and should require a minimal amount of testing to provide confidence in his predictions.

2. Evaluation of Test Techniques

It is beyond the scope of this document to present detailed guidelines for establishing the philosophy, objectives, and procedures for panel test programs. However, experience gained from the many tests that have been conducted during the past dozen years does provide insight into the choice of acceptable test procedures and indicates problem areas that may be avoided by careful planning. The suggestions and procedures presented in this section largely reflect the results of literature search, interviews, and data reduction that were accomplished in the formulation of the Criteria presented in Volume I.

The discussion encompasses two general classes of testing, viz. (a) research tests and (b) hardware development tests, both classes may involve vibration and/or wind tunnel phases of testing. The major distinction between the two is that research requires a physical model (test specimen) dictated more by mathematical formulation than by practical requirements, whereas a development test specimen usually simulates an actual structural design. The research test provides a physical verification of mathematical prediction and the development test provides assurance that a panel design is adequate for its intended use.

Instrumentation requirements vary with the test objectives; frequency and deformation strain measurements are easily obtained from strain gages bonded directly to a panel. Mode shape is more difficult requiring the output from an attached pickup (accelerometer, velocity pickup) which causes local inertia loading, or from a non-contacting sensor (inductive, capacitive). The state

of the art of non-contacting pickups is improving but there are still problems with calibration and output linearity. The most stringent requirements, however, usually occur for elevated temperature tests. The displacement and strain measurements that are made with relative ease during room temperature tests often dictate the procedures and goals of elevated temperature tests. Data from a high temperature specimen must be obtained either from special high temperature sensors or from room temperature sensors that are protected from the heat.

(a) Research Tests - The exacting requirements for research tests begin with the design of the specimen, including the support structure, which is intended to achieve the objectives of the test program. However, even though "ideal" physical conditions are not attained, the specimen should have measurable characteristics thus lending itself to mathematical description so that deviations from the "ideal" can be accounted for

The characteristic of skin panels that sets them apart from most other structures is the extremely small ratio of thickness to length (or to width) which causes a panel to have very little bending rigidity; the "membrane effect", defined here as the influence of inplane stress on lateral restraint stiffness (and hence on lateral vibration frequencies) is well known but is difficult to control. This effect may be caused by several conditions that can be controlled during the fabrication or test phases; aside from stresses built in during initial assembly the more common causes are slight initial curvature, differential pressure (unequal pressures on panel faces), and differential temperature (panel at a temperature different from the support structure). If such conditions are present and their magnitudes known, then their effects can be accounted for in analysis; otherwise, test results may appear spurious. A research test will usually require extensive instrumentation to insure that panel conditions are known and accounted for.

(1) Vibration Test - The purpose of vibration testing is to verify the structural assumptions in an analysis or to obtain modal characteristics for subsequent use in flutter analysis. It has been emphasized in this report that the structural portion of panel flutter analysis is usually not as accurate as the predictions of supersonic aerodynamic pressures; hence the need for good vibration data. Experience indicates that failure to properly control or account for the following items seriously jeopardizes the validity and interpretation of the test results:

Membrane stiffening effects (as discussed earlier).
Enclosed cavity behind the panel.
Inertia loading due to use of contacting type of exciter or pickups.
Large dynamic deflections (detected by frequency change that accompanies increases in vibration amplitude).

(2) Wind Tunnel Test - The wind tunnel portion of research testing presents unique design challenges. In addition to the panel characteristics discussed previously, a set of aerodynamic conditions often exists that may alter flutter speeds of panels. Some general precautions are listed below:

Use continuous flow tunnel (size panel so that it is not struck by reflected shock waves).
Measure flow conditions near the panel surface.
If vibration data are used, bench and tunnel conditions should be matched, if at all possible.
Measure (and account for, if possible) such flow induced properties as boundary layer, static pressure distribution along tunnel wall, aerodynamic heating.

The Mach number range that is least troublesome for wind tunnel panel flutter tests lies approximately between M = 1.4 and M = 2.2. At lower Mach numbers, wall static pressures, boundary layer buildup, reflected shocks and "rough" flow cause problems with data interpretation. At higher Mach numbers, aerodynamic heating becomes a deciding factor in data interpretation due to induced thermal compressive load; but this effect can be accounted for by the acquisition of additional data from thermocouples and from high temperature strain gages. Mach number does not scale in dynamic modeling, therefore data must be obtained at each Mach number of interest.

(b) Hardware Development Tests - Although the instrumentation, fabrication, and test requirements are not as demanding as for research tests, development tests must be well planned in order to provide the required verification of design integrity. Adequate simulation not only of the anticipated aerodynamic conditions but also of the anticipated structural condition of the panel are required if the test is to provide a realistic appraisal of the design. Fabrication of a test fixture from sections of an actual airframe provides the structural simulation of panel attachments and edges supports; however, there may be loadings generated in flight that must be simulated by artificial means. The following items are very important and should be high on the list of priorities for simulation of flight conditions:

Mach number.
Boundary layer (thickness should be simulated if at all possible).
Inplane loads on panel.
Different[illegible] pressure on panel.
Tempera[illegible] f panel.

With the singular exception of in-flight panel flutter research tests conducted by NASA Flight Research Center, other in-flight panel flutter tests have been performed as the final phase of hardware development, i.e., the final proof in the true aerodynamic environment. Generally, data is not taken unless

there are symptoms of panel flutter, in which case suspect panels are instrumented (strain gage or vibration pickup) until the fluttering panel(s) are located. A "fix" is made, the panel(s) reinstrumented, and the vehicle reflown through the previous flight condition; the adequacy of the "fix" is determined from the recorded data.

Many well managed test programs have been conducted and a study of the techniques employed can provide valuable guides in formulation of new tests. Different investigators have used varying means to account for test errors; for example, the cavity problem was solved in tests described in Reference 27 by opening the rear of the panel holder, while in Reference 17, vibration tests were conducted in a reduced pressure.

3. Evaluation of Published Panel Design Criteria

One may wonder why, with the large amount of work that has been done, the state of the art in the design of flutter-free panels is not further advanced from its present status. It has been stated that we have adequate understanding of the induced aerodynamic forces and we also have the capability for predicting the structural forces that come into play. However, we have continued to find discrepancy between theory and experiment and have apparently lacked the design tools to insure optimum panel design. The reason lies partly in the fact that the large number of variables that affect panel flutter boundaries are extremely difficult to handle; in addition, the failure of theory in predicting stability boundaries has discouraged many designers from using theoretical trend data in a design situation. This Section presents a discussion of the design techniques that are in current use.

a. Nondimensional Parameters

It is found, when nondimensionalizing certain parameters in panel flutter analysis, that it is convenient to present data in one of the following forms

$$\lambda = \frac{2q\ell^3}{\beta D}$$

or

$$\phi = \left(\frac{\beta E}{q}\right)^{1/3} \frac{t}{\ell}$$

These parameters are closely related, and by making the substitution

$$D = \frac{Et^3}{12(1-\nu^2)}$$

it can be demonstrated that

$$\phi^3 = \frac{24(1-\nu^2)}{\lambda}$$

or, if $\nu = 0.3$,

$$\phi^3 = \frac{21.84}{\lambda}$$

The parameter ϕ offers the advantage of presenting flutter boundaries that are directly proportional to panel thickness, while the term γ presents the data as a function of dynamic pressure. The panel designer might desire an all inclusive parameter that takes into account the many factors that affect flutter boundaries. Such a parameter has not been found; this is probably due to the complex nature of the phenomenon.

The fundamental criterion for designing panels might conceivably be based on:

(a) Determining a flutter fatigue life which a panel can endure without jeopardizing the vehicle mission, or

(b) Designing for complete avoidance of flutter instabilities.

Sufficient data are not available to use the first concept with any degree of confidence; therefore, the latter approach has been used almost exclusively throughout industry and is the basis for current specifications.

Currently used criteria naturally fall within the following categories:

(a) Analytical - based on theoretical concepts.

(b) Empirical - based on experimental data.

(c) Analytical/empirical - based on a combination of theory and experiment.

The purely analytical approach offers the convenience and economy of parametric studies, but suffers from a lack of comparison with measured data. The use of a purely experimental design approach instills more confidence in panel designers, but has an obvious disadvantage in that it is not feasible to obtain the wealth of data that would be required to specify all flight situations. A sensible mixture of theory and experiment offers the best procedures within the current state of the art. Several facilities were visited during the course of this effort; in formulating design criteria, most of the individuals who are charged with panel design combine theoretical and experimental data.

Most of the panel flutter design criteria that are in current use can be traced to experimental origins. Furthermore, the following documents have contributed heavily to the basic information that has been used for design:

NASA TN D-451

NASA TN D-1386

Langley Working Paper LWP-177.

In view of the amount of work that has been done in panel flutter and related fields (see Bibliography), it is surprising that so few documents have provided such a broad basis for current design efforts. At the same time the trend toward dependence on experimental data accentuates the lack of confidence in theoretical data which has been caused by the lack of correlation between theory and experiment.

b. NASA TN D-451

This document presented the results of an early organized attempt to provide comprehensive panel flutter information. The results from both wind tunnel tests and in-flight occurrences of panel flutter were presented in the form of the panel flutter parameter Φ versus ℓ/w. The upper limit of the data was enclosed by an envelope that has come into wide usage for panel design. The data for unstiffened panels is reproduced in Figure 1. A theoretical analysis for the flutter of orthotropic panels was also presented, but the authors stated that experimentally determined boundaries should be relied on for design information for all but the simplest configurations.

The experimental work was presented to show the status of the panel flutter problem. Although the authors did not suggest that the envelopes be used as a basis for design, the results from TN D-451 were seized immediately and applied to the design and diagnosis of skin panels. It is historically significant that the report became available at a time when aircraft companies were able to use the data to diagnose and remedy panel flutter problems that were encountered on early supersonic airplanes.

c. NASA TN D-1386

This report describes an investigation of the effects of compressive stress on the flutter of panels. Aluminum and steel panels were fluttered with varying axial compressive loads; it was found that the minimum dynamic pressures at flutter occurred when the panels were loaded to a point near the calculated critical buckling stress. An important conclusion was that the envelope of TN D-451 might be unconservati if a panel were subjected to inplane stress. The buckled panel boundary from TN D-1386 is shown in Figure 2 and is compared with the flat panel boundary from TN D-451.

d. NASA Langley Working Paper LWP-177

This document is an interim report of work that was conducted on a series of flat panels ranging from $\ell/w = 1.0$ to 4.5 at Mach numbers 1.56 and 1.96. The panels were stressed longitudinally; therefore, the program offers experimental panel flutter boundaries as a function of inplane stress and length-to-width ratio. The test apparatus incorporated

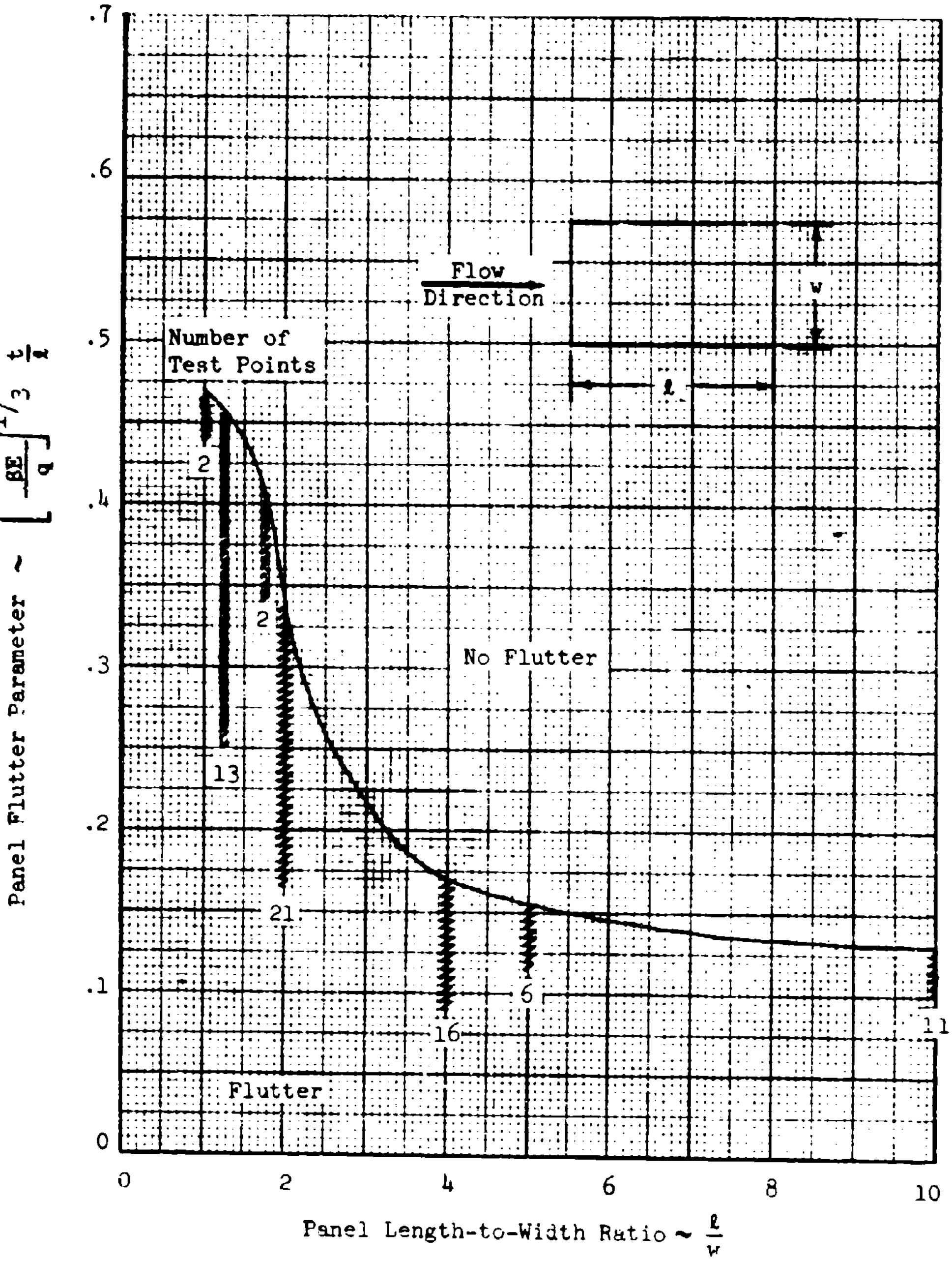


Figure 1 - Flutter Boundary for Unstiffened
Panels from NASA TN D-451 (Reference 26)

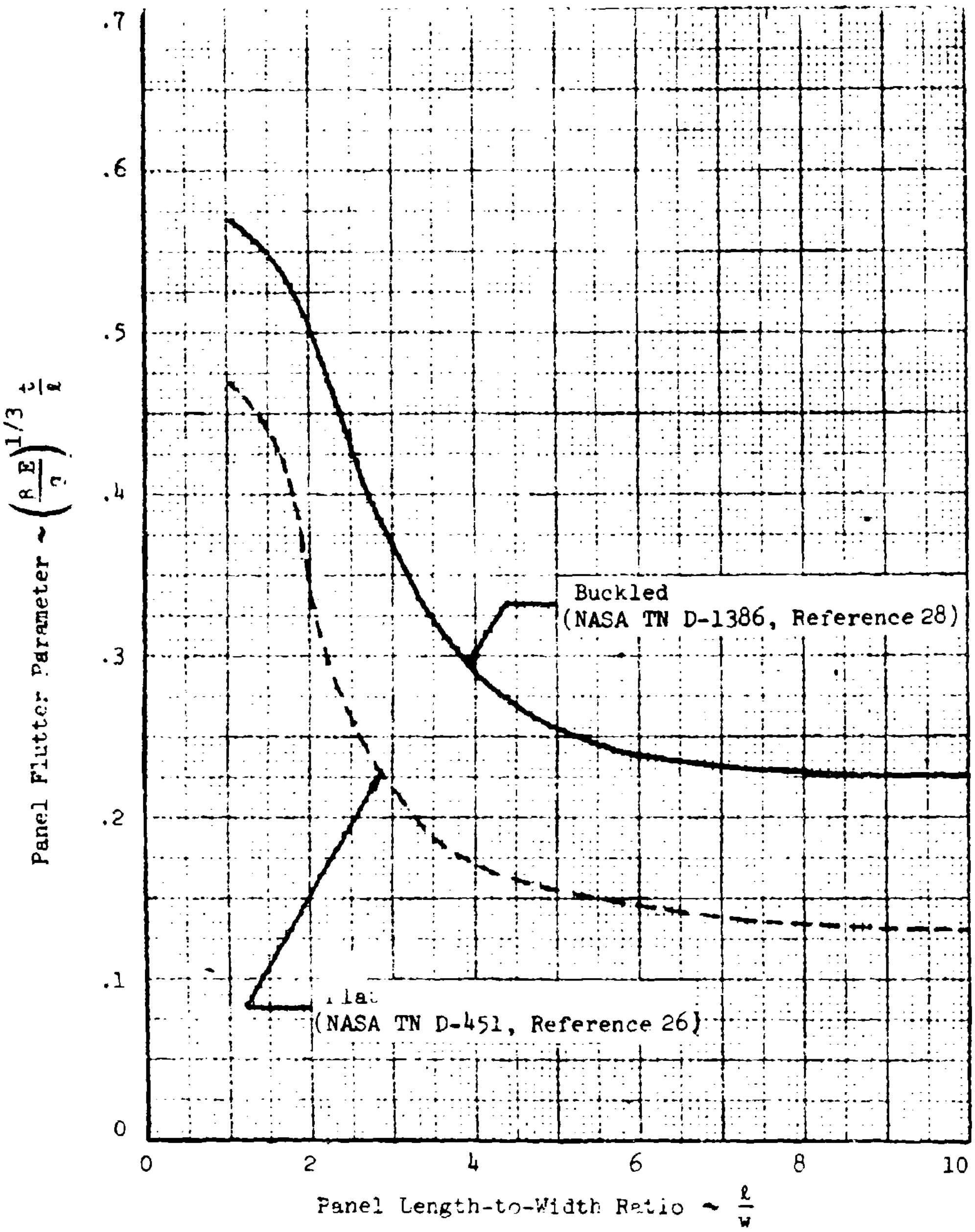


Figure 2 - Comparison of Buckled and Unbuckled Panel Flutter Boundaries

features that had been found to be desirable in earlier test work. At the time of this writing, the final report had not been released, but it was felt that the data in LWP-177 was the best available for a flat, stressed panel. The zero stress and buckled panel flutter parameters from LWP-177 are shown in Figure 3. The $M = 1.96$ data has been chosen as the basis for the design of flat panels in the range $1.0 \leq \ell/w \leq 4.5$.

The following two documents have been prepared with the specific goal of assisting in the design of skin panels and present criteria for the prevention of panel flutter:

e. ARTC Report No. ARTC-32

In 1960, Panel 58-A (Dynamics and Aerolasticity Research) of the Aerospace Industries Association assigned to McDonnell Aircraft Corporation the responsibility for gathering data on in-flight incidences of panel flutter for the purpose of improving the state-of-the-art. The assignment resulted in a set of design criteria that encompassed flight data, wind tunnel data and literature survey. Based almost exclusively on empirical data, ARTC-32 presented Mach number, inplane stress, sweep, and buckling corrections. It was recommended that a concentrated attack should be directed back to the fundamental research problem concerning the flutter of a flat, unbuckled, rectangular, uniform thickness panel. Furthermore, the report, which was released in 1962, pointed out the limitation in the use of β in the panel flutter parameter. Advances that have been made in theory and experiment since that time have provided designers with a substantially greater amount of background information.

f. NASA Space Vehicle Design Criteria

In 1964 the National Aeronautics and Space Administration circulated a set of criteria, designated NASA SP-8004, for design of space vehicles. As a design document, the criteria were to be regarded as guidelines and not requirements. Panel flutter was covered in a section dealing with structures, where it was recommended that panels be designed to withstand dynamic pressures up to 1.5 times the maximum anticipated flight value. Furthermore, it was recommended that tests should be conducted on at least one panel of each structural type to include flow angularity, local Mach number, local dynamic pressure, thermal and mechanical loads and differential pressure. Recommended practices were presented for selecting critical panels both with and without midplane stress. It was recommended that TN D-451 not be relied upon for $M_L < 1.5$. Reference was made to twenty-one published reports for background information and for detailed design data.

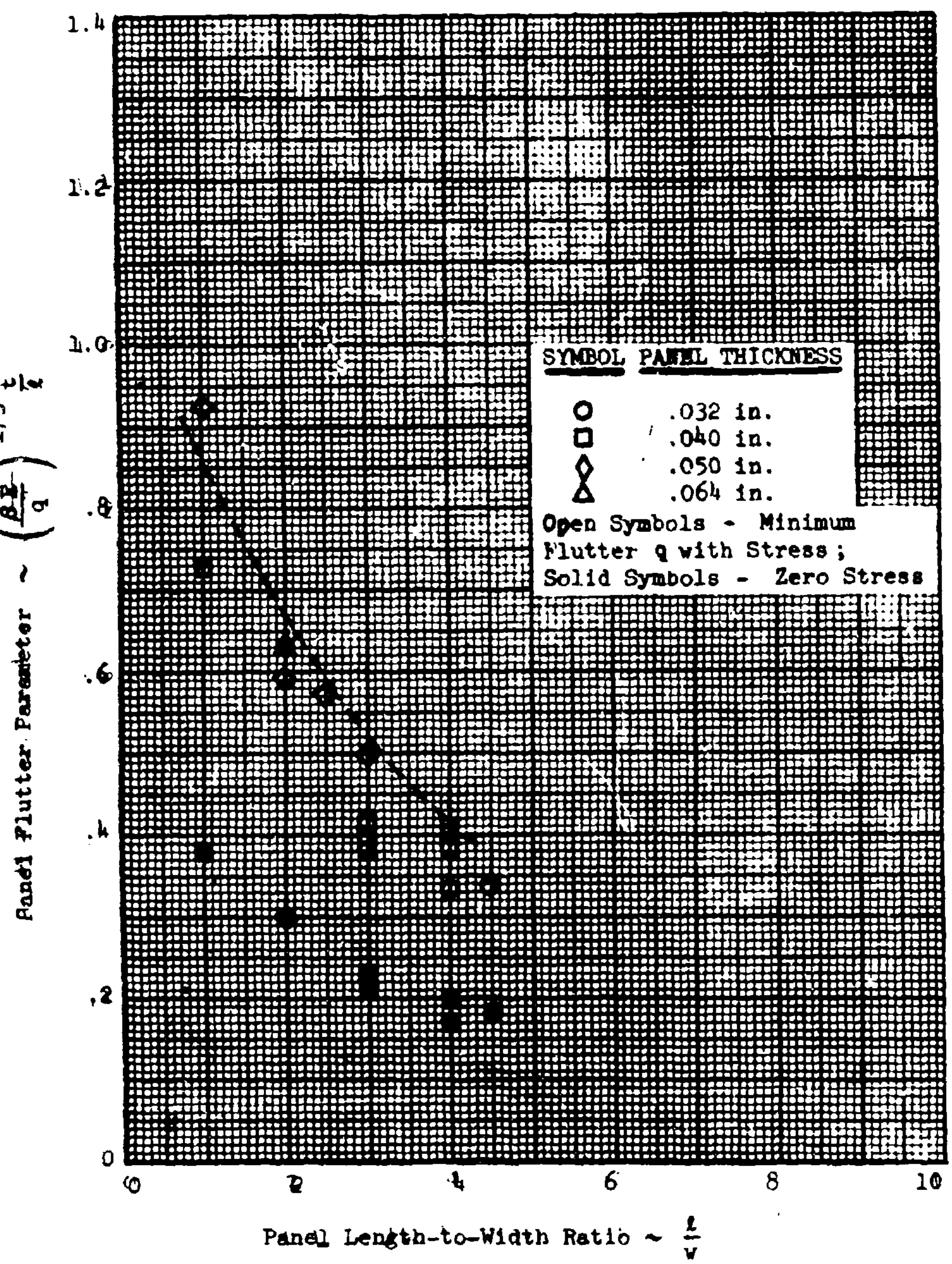


Figure 3 - Experimental Flutter Boundaries from LWP-177 (Reference 17)

SECTION III

PRESENTATION OF DATA AND TREND STUDIES

This Section presents the results of studies and data correlations that pertain to the parameters that are included in the design criteria. In addition, the utilization of the data trends in the final design presentation is discussed for each parameter.

1. Dynamic Pressure (q)

Dynamic pressure ($q = \frac{1}{2} \rho V^2$) determines the level, or intensity of the aerodynamic forcing function, and stability boundaries are often specified in terms of critical dynamic pressure. It is common in some branches of aeroelasticity to refer to stability boundaries in terms of velocity (i.e., flutter speed), especially if aerodynamic damping (which varies with V) has a strong influence on the instability. It will be noted later in this report that damping effects on panel flutter are poorly defined; therefore, dynamic pressure has the greater significance. In fact, dynamic pressure, q, appears directly in the nondimensional panel flutter parameter, Φ, that describes the flutter level of panels.

2. Mach Number (M)

Point function aerodynamic theories are applicable under certain conditions in supersonic flow (Reference 9). They have a local pressure-slope relationship that is approximated by

$$p = -\frac{2}{\beta} \left(\frac{\partial w}{\partial x}\right) q$$

in which $2/\beta$ is the lift curve slope and $\partial w/ \partial x$ is the instantaneous inclination of the surface to the airstream. Since $\beta = \sqrt{M^2-1}$, this equation cannot be expected to hold when $M \to 1$ because $2/\beta \to \infty$. This situation has contributed to the confusion that has dominated transonic panel flutter problems, and has left the designer with an untenable situation if he uses β indiscriminately in the panel flutter parameter. The Mach number range 1.1 thru 1.6 has produced most of the in-flight occurrences of panel flutter, and also the greatest analytical difficulty. It was decided that a Mach number correction factor, to replace β in the transonic region, must be obtained largely from experimental data.

The effect of Mach number on flutter speed has been studied both experimentally and theoretically. The results of those studies have been analyzed and are presented, for panels of $\ell/w = 1/2$ and 2, in Figures 4 and 5. The overwhelming majority of transonic panel flutter data (References 16 and 27) have been obtained with panels of these length-to-width ratios. The data shown on these plots are normalized to the critical dynamic pressures at Mach 2; this value was used in order to minimize the effects of other parameters (aerodynamic heating for $M>2$; damping and/or boundary layer for $M<2$) that might obscure the Mach number trend. It also provides a convenient tie-in with the M = 1.96 data of Reference 17. The solid line on Figure 4 envelopes the $\ell/w = 1/2$ data below M = 2. When compared with the $\ell/w = 2$ data of Figure 5,

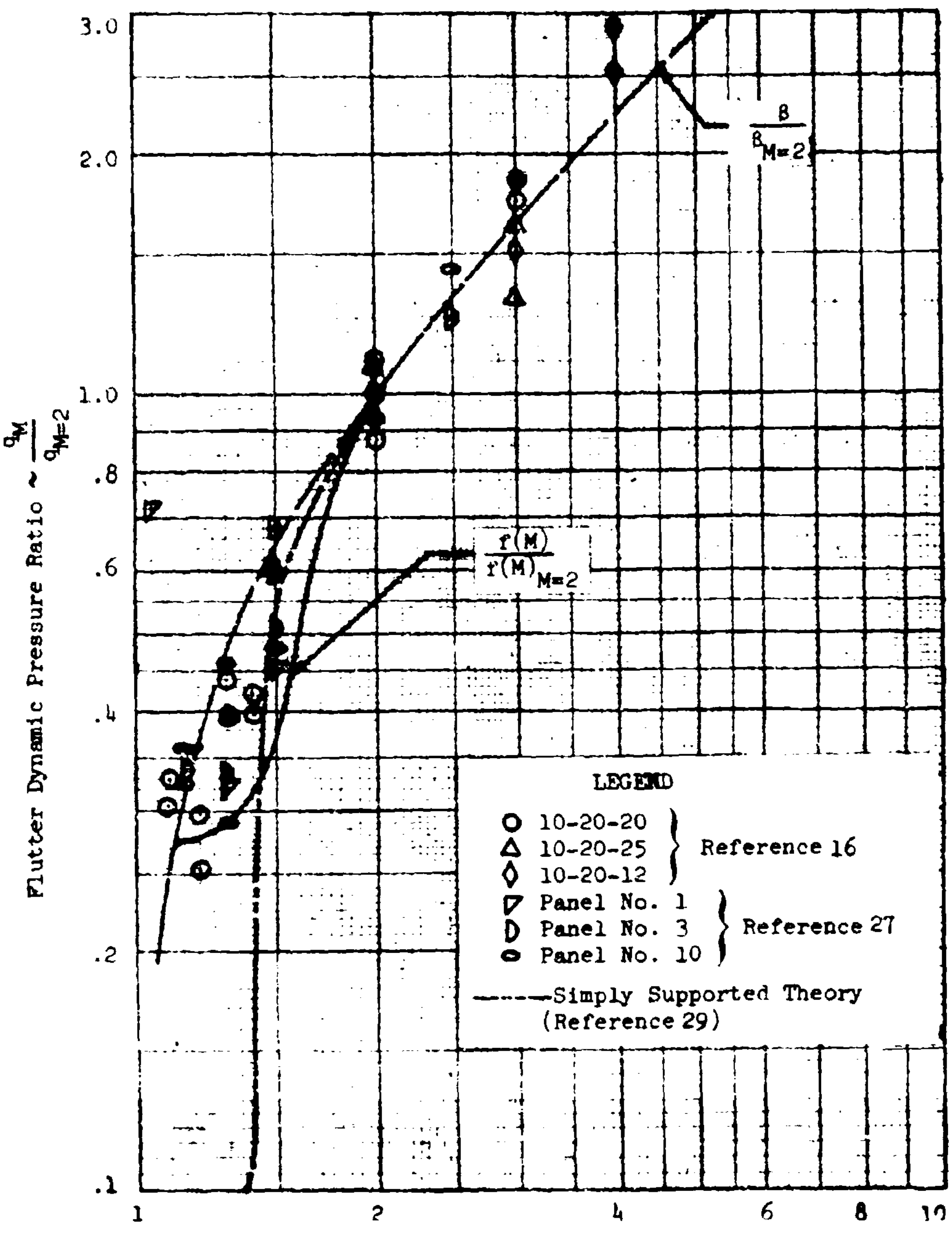


Figure 4 - Flutter Dynamic Pressure Ratio Versus Mach Number for Length-to-Width Ratio of One-Half

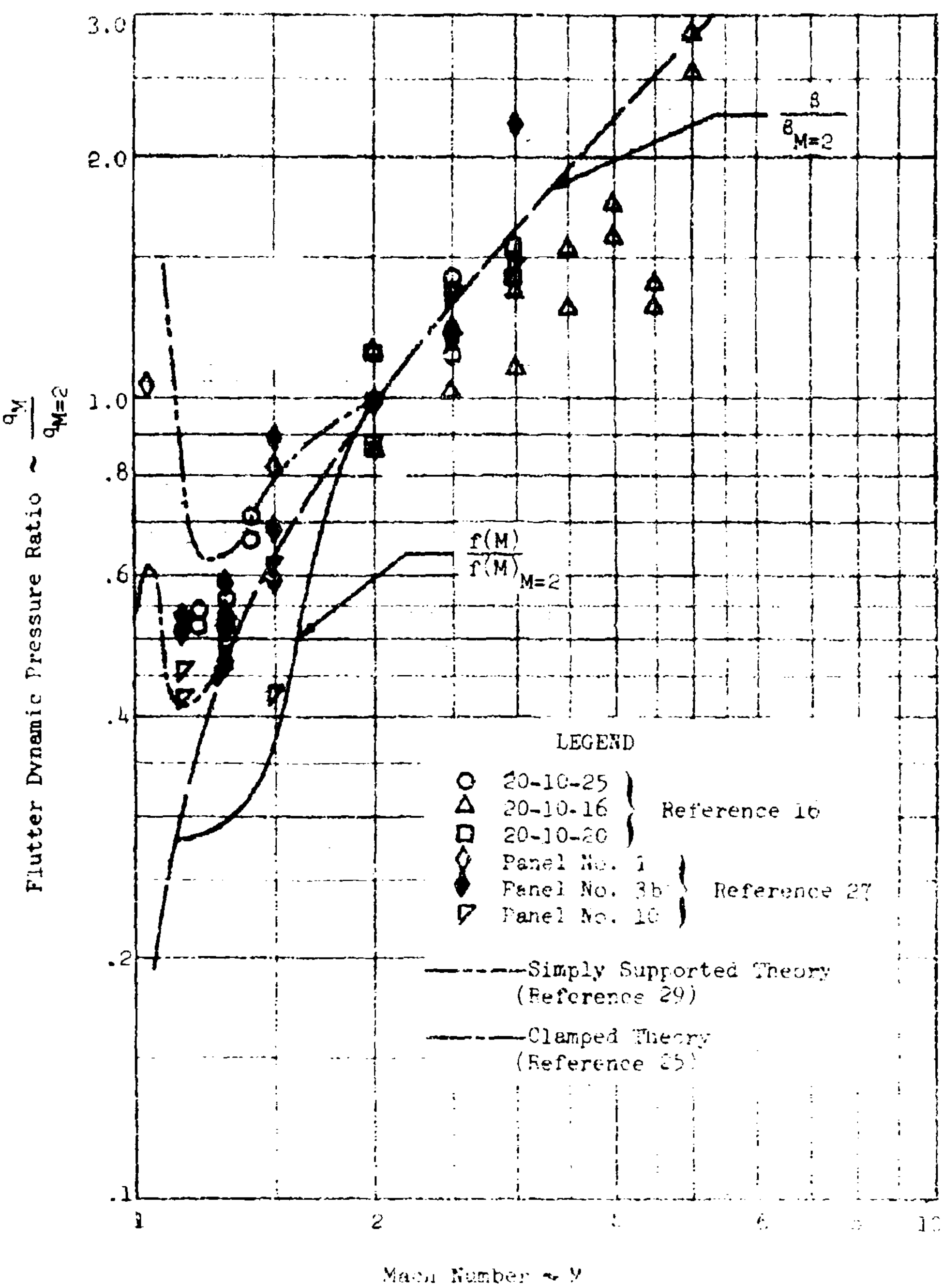


Figure 5 - Flutter Dynamic Pressure Ratio Versus Mach Number for Length-to-Width Ratio of Two

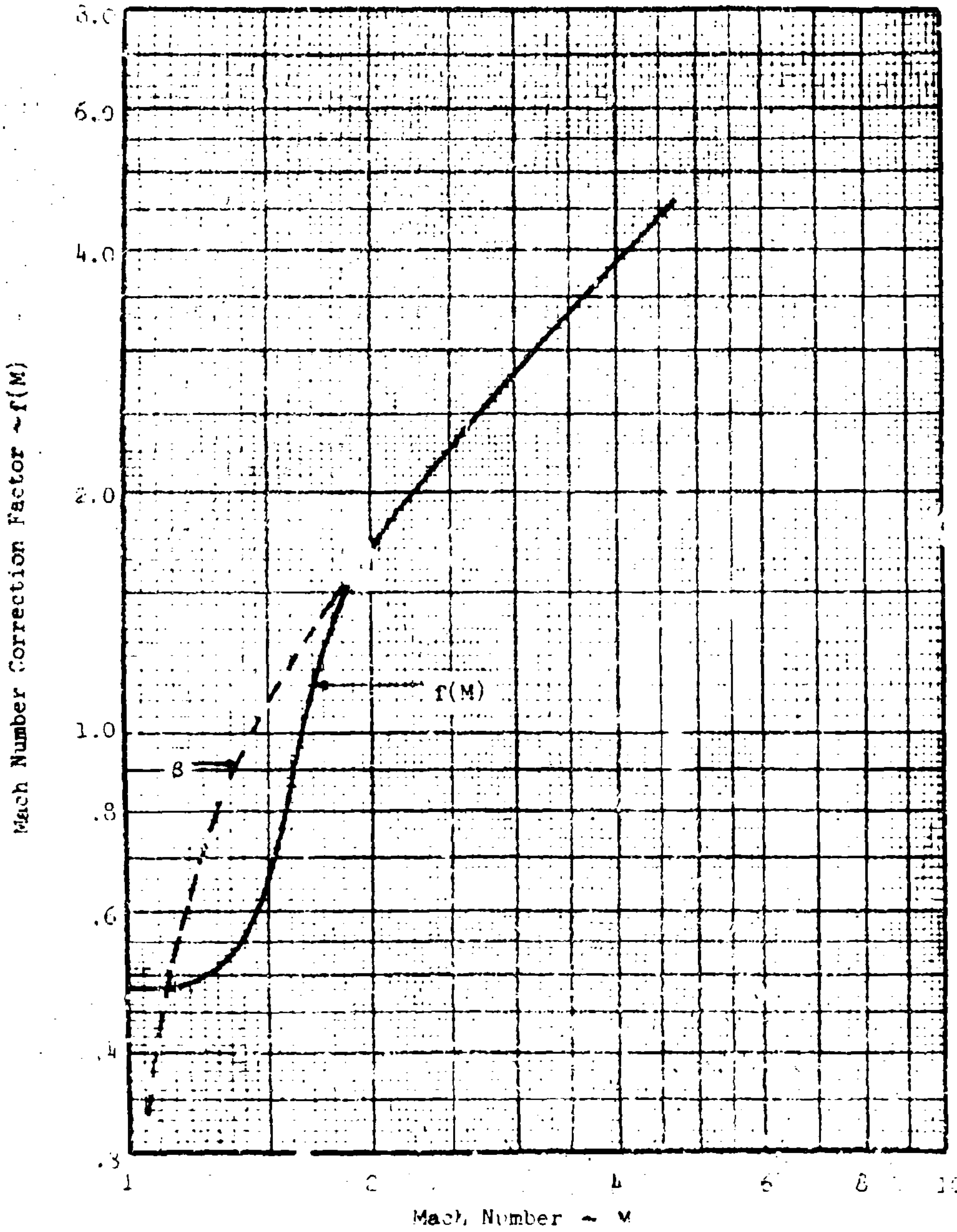


Figure [illegible] - Mach Number Correction Factor Versus Mach Number

the envelope shows that there is an ℓ/w effect in this region. The spread in the data for M>2 in Figure 5 is attributed to aerodynamic heating and is not considered useful for modifying β. Theoretical results using a three-dimensional, linearized, potential flow aerodynamic theory are also given for comparison with experiment. The correlation obtained with an idealized simply-supported panel (Reference 29) is poor but a clamped panel analysis (Reference 25) is in good agreement with test data at $\ell/w = 2$. The flutter speeds that are predicted in transonic analyses are very sensitive to the level of structural damping which may account for some of the discrepancy between theory and experiments.

Sufficient data are not yet available to define the transonic boundary trends as a function of ℓ/w; therefore the more conservative of the data envelopes has been chosen as the required Mach number correction trend for all values of ℓ/w. The Mach number correction factor f(M) is shown in Figure 6 in comparison with β. These curves were obtained by multiplying the $f(M)/f(M)_{M=2}$ and $\beta/\beta_{M=2}$ curves shown in Figure 4 by the normalizing factor ($\beta_{M=2} = \sqrt{3}$) used in their preparation.

3. Angle of Attack

The flow conditions (Mach number and dynamic pressure) due to the component of stream velocity that is parallel to the plane of the panel, are called local conditions and have a strong influence on flutter speed. If the panel is aligned to the free stream velocity so that the free stream velocity vector is parallel to the plane of the panel, the free stream and local conditions are the same. However, if the panel is inclined to the free stream, significant differences can occur between the local and free stream flow conditions. Additionally, a panel that is inclined to the airstream will experience a static aerodynamic pressure acting on its surface.

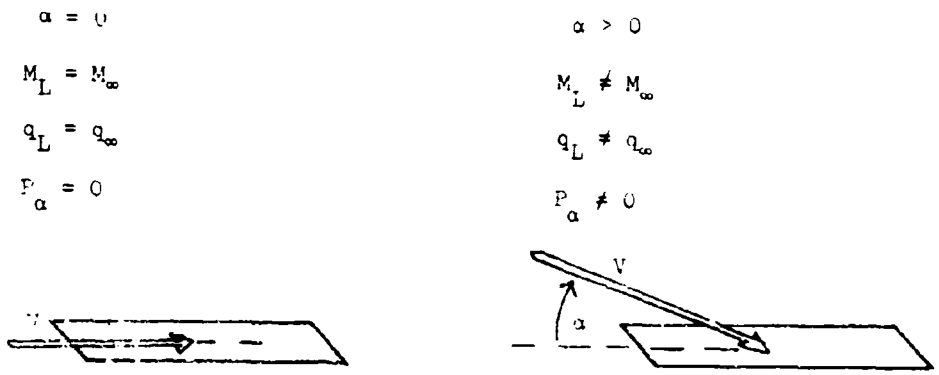


Figure 7 - Sketch Showing Relationship between Free Stream and Local Flow Conditions

The sketch shown in Figure 7 illustrates the features of the flow characteristics. Relationships between the local and free stream parameters can be obtained from sources such as Reference 30. The available relationships include $(q/\beta)_L/(q/\beta)_\infty$, M_L/M_∞, and $q/_L/q_\infty$.

With the presence of the static overpressure p_α, it is not at all clear that a panel design will be determined by panel flutter. If the designer is in doubt, he may run a check by the following procedure:

(a) Determine $\left[q/f(M)\right]_L$.

This is accomplished by using free stream conditions, M_∞ and q_∞ together with α to obtain the ratio $(q/\beta)_L/(q/\beta)_\infty$ and the local Mach number M_L. Then computing $\beta_L = \sqrt{M_L^2-1}$ and determining $f(M_L)$ from Figure 6, the expression

$$\left(\frac{q}{f(M)}\right)_L = \left(\frac{q}{\beta}\right)_L \times \frac{\beta_L}{f(M_L)}$$

can be evaluated throughout the anticipated flight path. The maximum value of $q/f(M)$ (free stream or local) is used in determining the final panel thickness.

(b) The differential pressure resulting from the inclination is calculated, and thickness correction made as described in Section III of Volume I.

The designer is cautioned to recall, in using q/β, that $\beta \to 0$ as $M \to 1$.

4. Length-to-Width Ratio (ℓ/w)

The length ℓ and width w are the flow oriented dimensions of a rectangular panel. The convention used in this report is shown in the following figure.

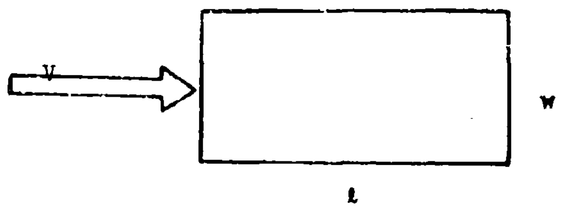


Figure 8 Designation of the Sides ℓ and w

The ratio of the lengths of the sides, ℓ/w, plays an important role in panel flutter.

One pronounced effect of variation in the length-to-width ratio concerns flutter mode shapes and modal participation therein. The lateral response of fluttering panels usually follows an established pattern in which the largest motion (and failure) occurs at the trailing edge of the panel. Typical mode shapes are sketched in Figure 9 for low and high ℓ/w panels.

V

(a) 

Low ℓ/w

V

(b) 

High ℓ/w

Figure 9 Typical Flutter Mode shapes for Low and High ℓ/w Panels

From the analytical standpoint, the low ℓ/w panel is easier to analyze. It is obvious that the waveform in Figure 9 (a) can be described with considerably fewer Fourier components than the wave form of Figure 9 (b). This fact is directly related to the number of vibration modes that must be included to obtain a satisfactory flutter solution. It has been shown in Reference 24 that while 2 to 4 modes may yield satisfactory flutter speeds for panels with low ℓ/w ($0<\ell/w<3$), as many as 60 modes may be required for $\ell/w = 10$. The authors prefer modal solutions for investigating parametric variation in factors of interest; however, the supplementary analyses were found to give doubtful answers for $\ell/w>4$. Therefore, the thickness correction factors presented as a part of the design criteria do not extend beyond $\ell/w = 4$. The theoretical behavior of flutter boundaries at large ℓ/w can best be seen by modifying the baseline curve so that increasing ℓ can be studied independently.

A plot of $\Phi \times \frac{\ell}{w} = \left(\frac{f(M)E}{q}\right)^{1/3} \frac{t}{w}$

versus ℓ/w shows that the new parameter apparently approaches an asymptote of about 0.74. The significance is, of course, that panels of very large ℓ/w (say $\ell/w>5$) can be designed according to

$$\Phi \times \frac{\ell}{w} = 0.74$$

The inverse of this relationship was given in Reference 18.

A second effect of ℓ/w involves a Mach number-ℓ/w interaction that has been previously noted in Reference 31, for example. The mechanism, occurring in the transonic flow regime is not nearly so well understood. Sufficient data are not available to delienate the M-ℓ/w interaction.

In this, as well as most other panel flutter criteria, ℓ/w has been used as a fundamental parameter in the design approach. If flexibility in design or redesign allows variation in ℓ/w, then this parameter can be used very profitably to save weight. Some guidelines for ℓ/w are

(a) For $\ell/w>1$, yaw correction not required

(b) For $\ell/w>5$, it appears that for a constant width the same thickness may be used for a panel with a given dynamic pressure, q. (Theoretical, requires experimental verification)

(c) A rectangular unswept panel is more stable when its shorter dimension is parallel to the air stream.

5. Flow Angularity

Flow angularity (also called yaw, or sweep) is an external aerodynamic condition whereby the airstream velocity vector is oriented at some angle Λ to the principle axis of the panel. In this report, Λ is measured in respect to a line that is parallel to the side ℓ; in order to prevent ambiguity the sweep angle is restricted to the range $0\leq\Lambda<45°$. The convention is shown in Figure 10.

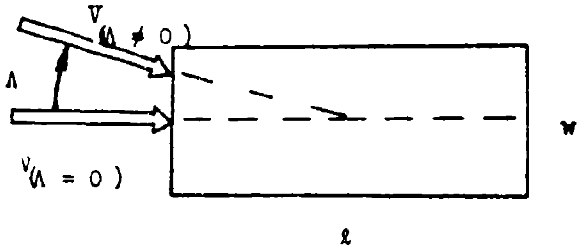
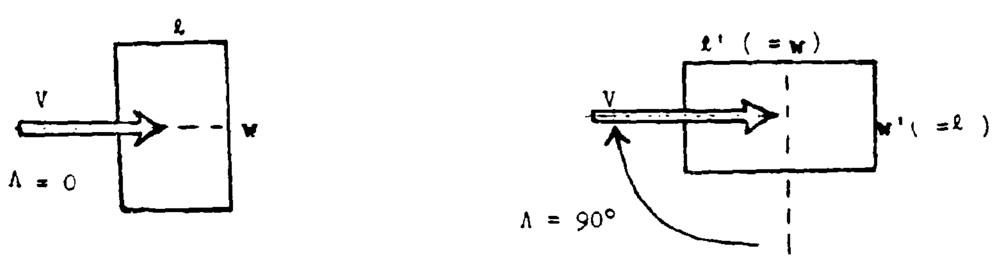


Figure 10 Orientation of Wind Direction Relative to Panel Demonstrating Flow Angularity

Several investigators have used supersonic aerodynamic theory, together with a component velocity flow approach to assess the effect of flow angularity. The results, as shown in Figure 11 (from Reference 32) indicate that sweep is stabilizing when $\ell/w > 1$ but may be strongly destabilizing when $\ell/w < 1$. Additional verification of the trend for $\Lambda = 90°$ was obtained from the baseline curve presented in Figure 36 of Section IV. Assume for any given value of ℓ/w, that the baseline parameter Φ_B represents the $\Lambda = 0$ flutter condition. Then we wish to relate to the same panel configuration that is turned 90°. Relationships are developed from the sketches in Figure 12.



$\phi_B(\frac{\ell}{w})$ $\qquad$ $\phi_B(\frac{\ell'}{w'}) = \phi_B(\frac{w}{\ell})$,

Figure 11. Flutter Boundary Relationship for a Panel at $\Lambda = 0$ and $\Lambda = 90°$

From the two relationships

$$\Phi\left(\frac{\ell}{w}\right)\bigg|_{\Lambda = 0} = \left(\frac{f(M)E}{q(\Lambda = 0)}\right)^{1/3}\frac{t}{\ell}$$

$$\Phi\left(\frac{w}{\ell}\right)\bigg|_{\Lambda = 90°} = \left(\frac{f(M)E}{q(\Lambda = 90)}\right)^{1/3}\frac{t}{w}$$

(Note that the latter equation is obtained from $\ell' = w$ and $w' = \ell$) and assuming that only the Φ and q change, the simultaneous equations give

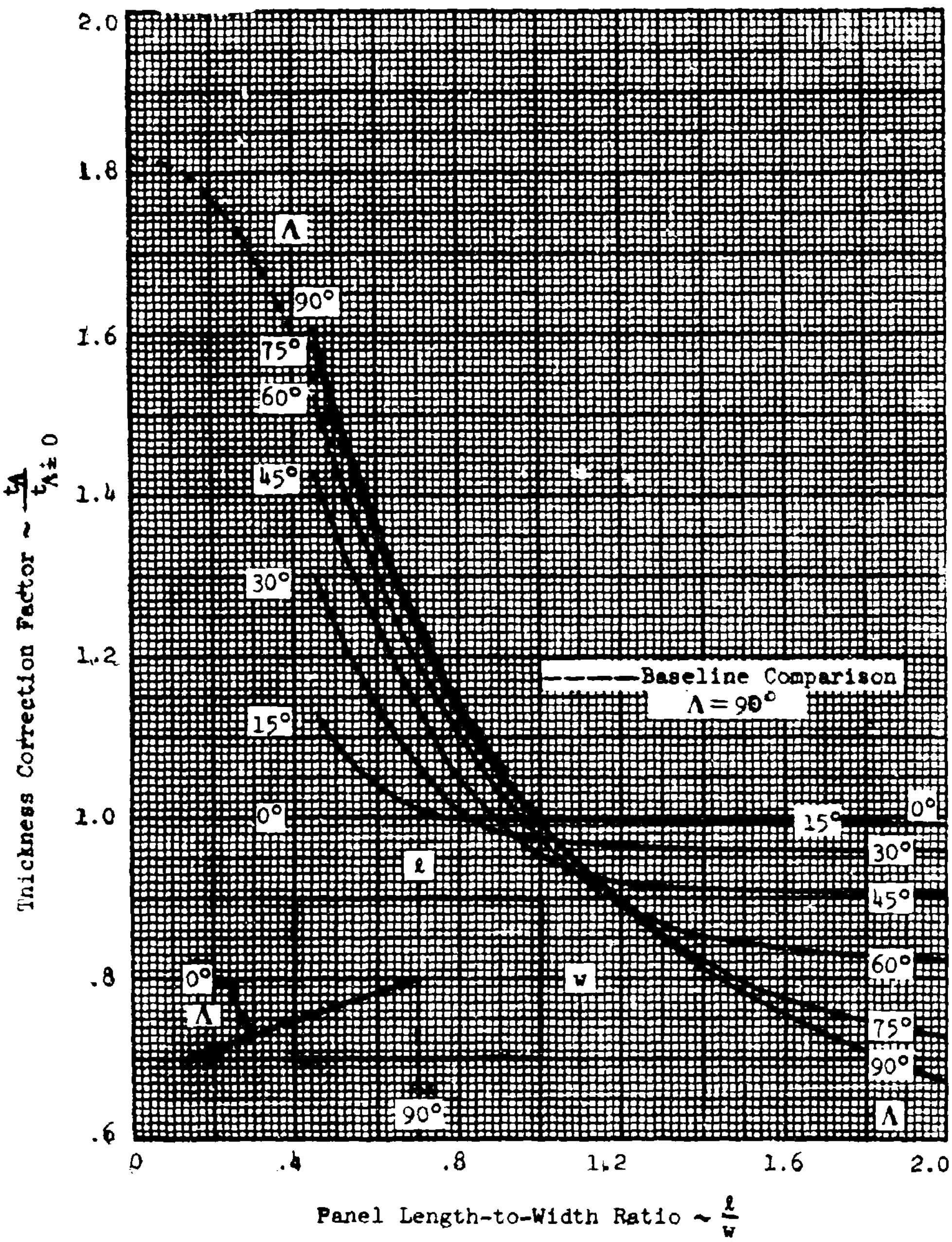


Figure 12. Theoretical Thickness Correction Factor Due to Yaw from NASA TN D-1156 (Reference 32)

$$\frac{q_{\Lambda = 90}}{q_{\Lambda = 0}} = \left[\frac{\phi(\frac{\ell}{w})}{\phi(\frac{w}{\ell})} \times \frac{\ell}{w} \right]^3$$

Therefore, assuming as we have that the baseline design curve is correct, then the last equation allows us to relate the $\Lambda = 0$ and $\Lambda = 90°$ flutter dynamic pressures. These data have been added to Figure 11 and verify the analytic trends, at least for $\Lambda = 90°$.

Seemingly, this would wrap up the sweep effect and we should need only to formulate the thickness correction factors t_Λ/t_B. However, experimental data from Reference 27 for $\ell/w = 1/2$ and $\ell/w = 2$ indicate that there is a Mach number effect on sweep that is not accounted for by the supersonic theory. The data are shown in Figures 13, 14, and 15 for $\Lambda = 30°$, $60°$ and $90°$, respectively. It is noted in every case that the yaw data points vary from approximately the theoretical curve down to no change at all. Inasmuch as the effect of Mach number could not be clearly defined, the conservative branches of the curves (that is, the greater thickness requirement) were chosen for criteria that account for flow angularity. The thickness correction curves are given in Figure 16 for $\Lambda = 15°$, $30°$, and $45°$ and interpolation may be used for intermediate values. These curves clearly indicate that thickness corrections for sweep should be made only for $\ell/w<1$.

6. Edge Conditions

The baseline panel is uniform, flat, unstressed, and is assumed to have edges that are completely restrained against rotation at the supports. There were two main reasons for using the clamped edge condition. In the first place, the best available flat panel data were obtained from a panel with edges that simulated the clamped condition. Secondly, the methods that are commonly used to install skin panels more nearly simulate clamped supports than simple supports.

Attempts have been made to design test hardware to simulate a simply supported configuration (no restraint against edge rotation), but this ideal condition cannot be approached as easily as the clamped configuration. The edge support correction that is presented here is based on theoretical data from Reference 23 and relates a required panel thickness for simply supported edges to the thickness required for the baseline panel (clamped edges). The curve t_{ss}/t_B versus ℓ/w is shown in Figure 17. Some judgment is required to estimate the degree of edge fixity; it is recommended that the designer assume clamped edges (i.e., no thickness correction) if panels are to be attached by continuous welds or closely spaced rivets. The designer should use a thickness correction if, in his opinion, the manner of attachment offers considerably less than rigid rotational restraint at the panel edges.

7. Curvature

The analysis described here applies to panels that are cylindrically curved in one direction and are constrained on all edges so that inplane motion is not allowed. The frequency equation (developed in

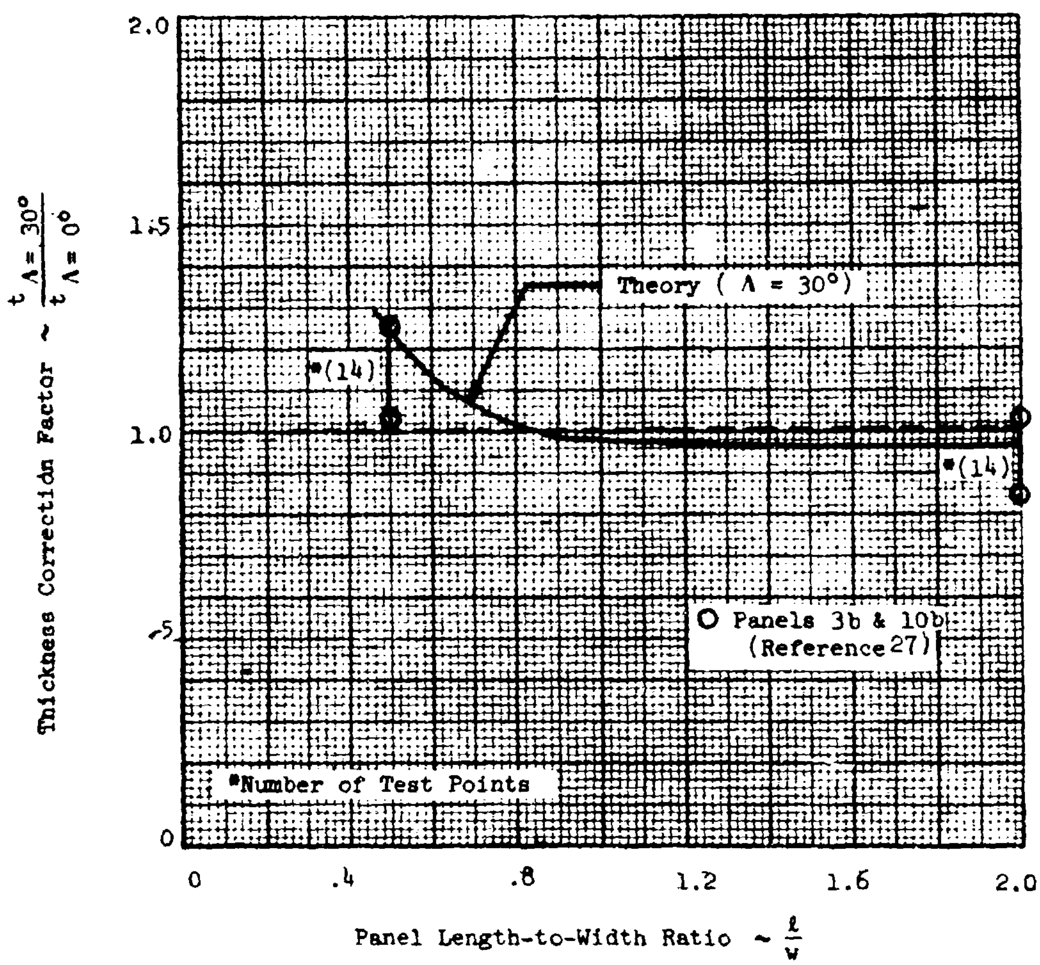


Figure 13 - Comparison Between Theoretical Thickness Correction Factor Due to Yaw and Experimental Data (Λ = 30°)

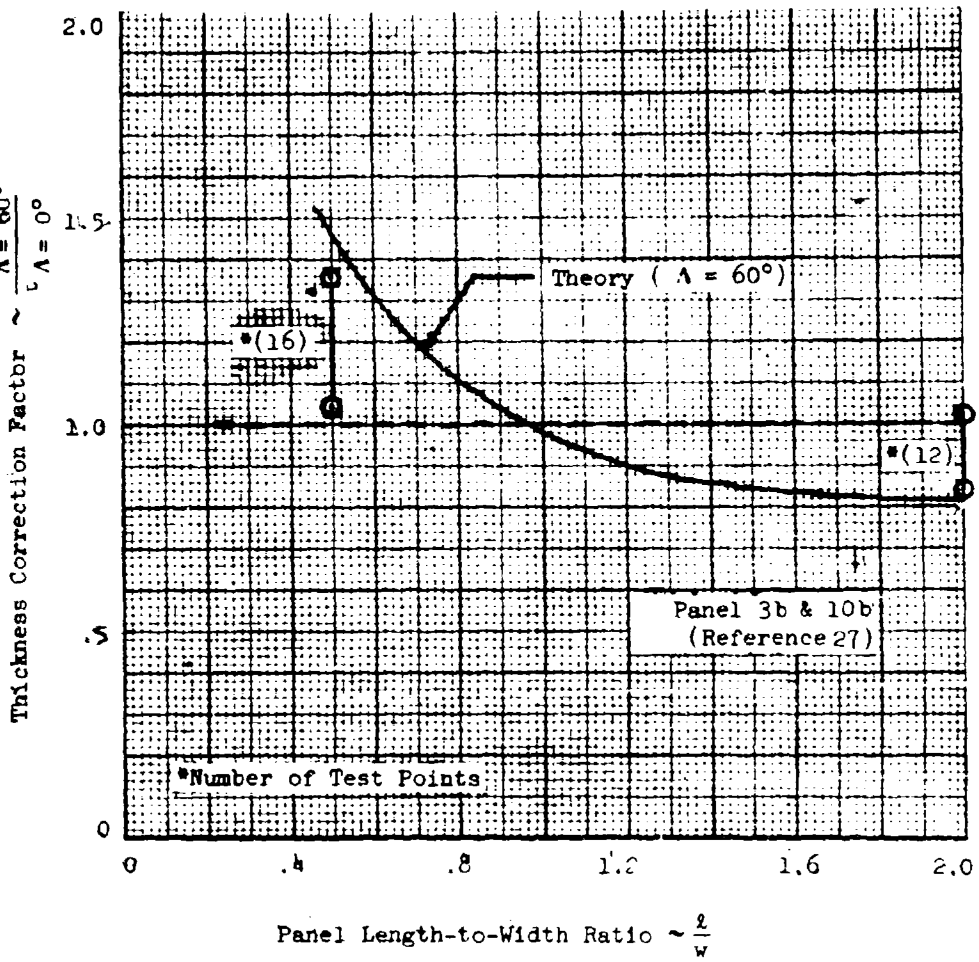


Figure 14 -Comparison Between Theoretical Thickness Correction Factor Due to Yaw and Experimental Data (Λ = 60°)

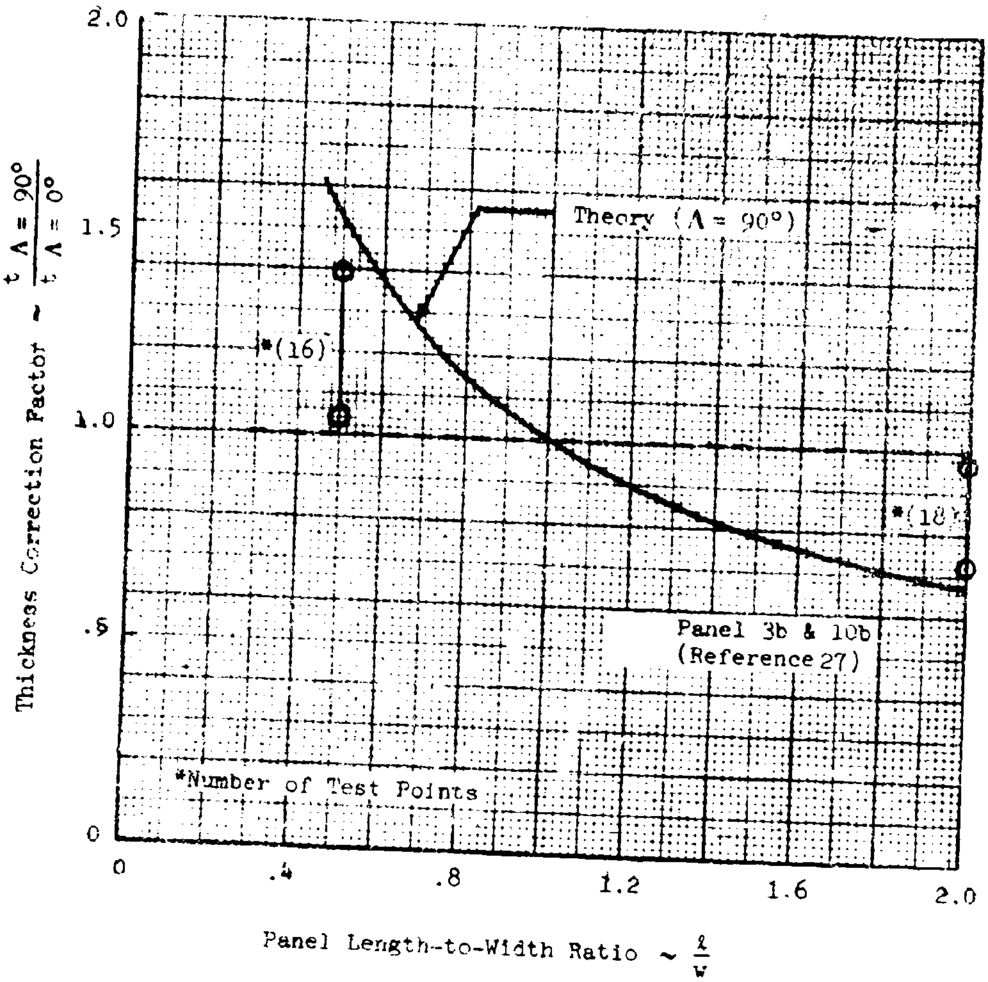


Figure 15 - Comparison Between Theoretical Thickness Correction Factor Due to Yaw and Experimental Data (Λ = 90°)

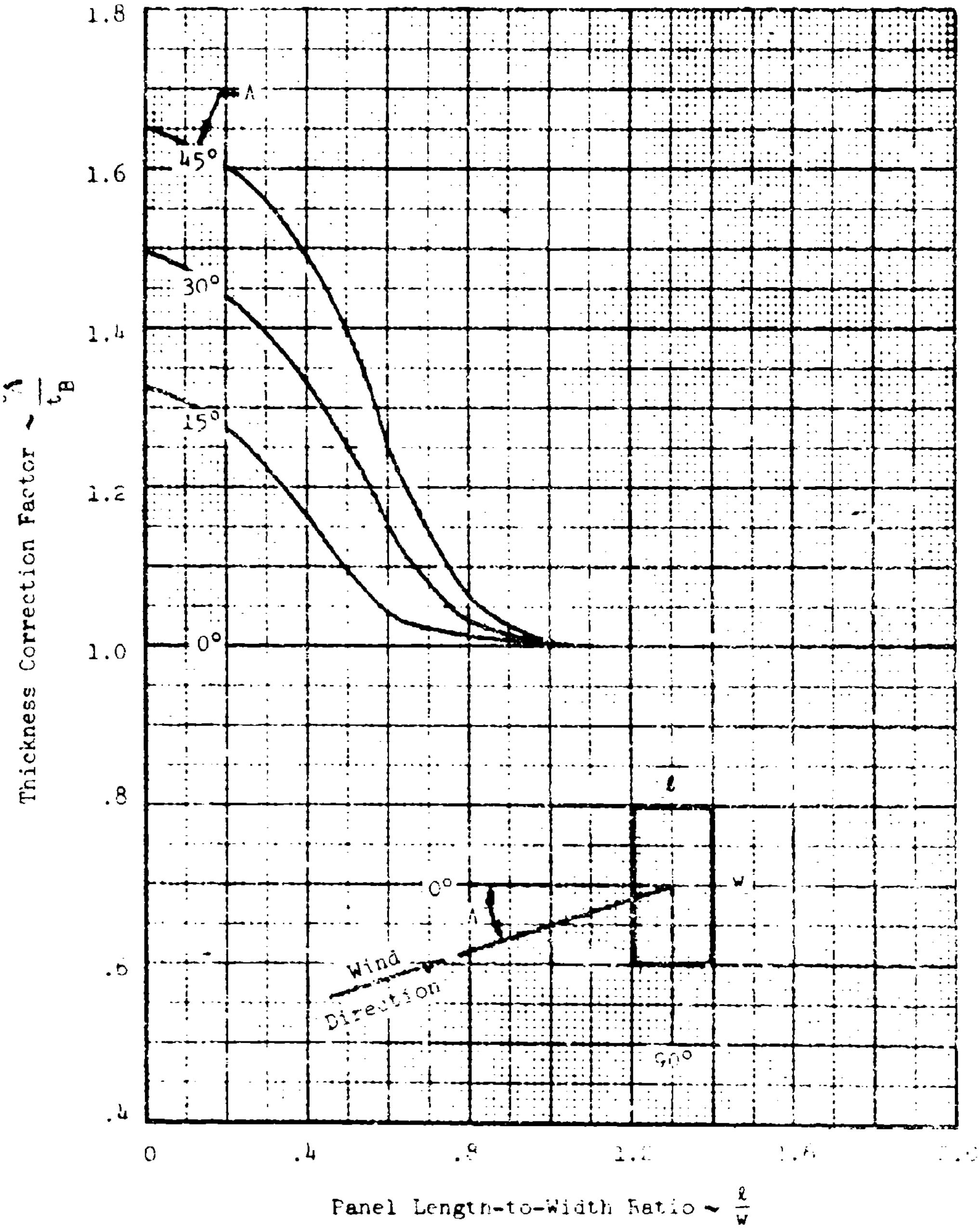


Figure 16 - Thickness Correction Factor for Flow Angularity

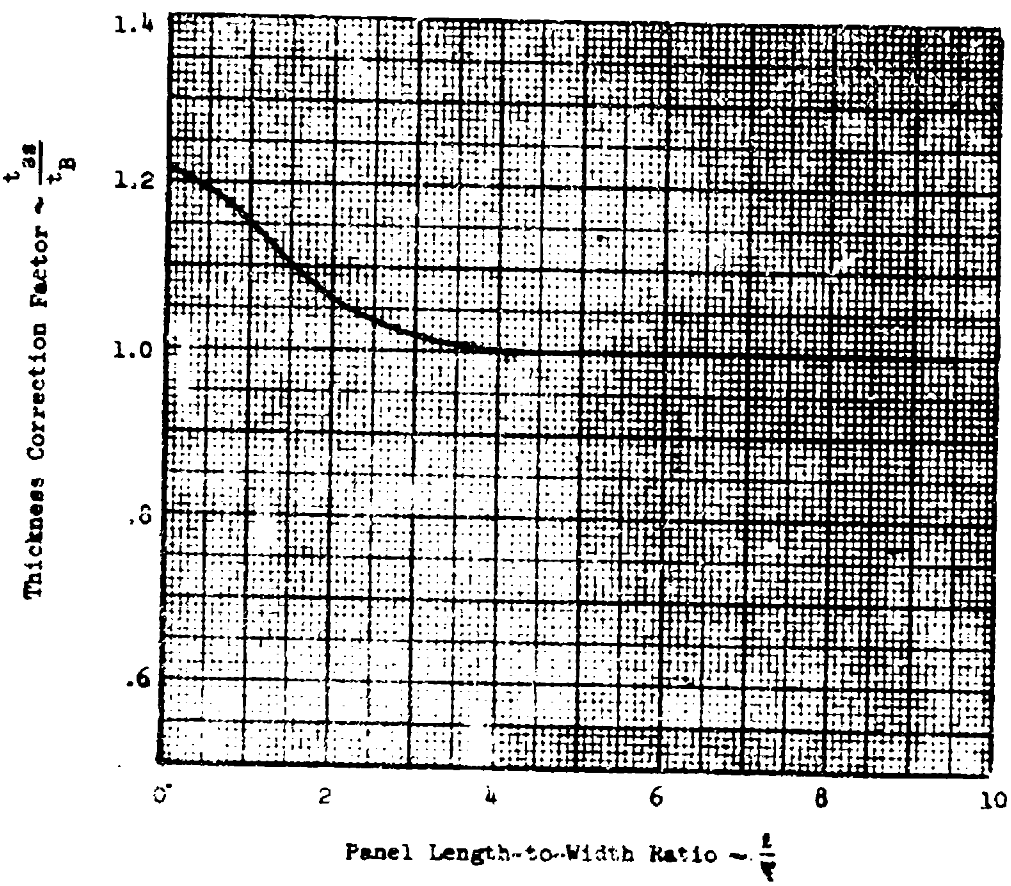


Figure 17 - Thickness Correction Factor for a Simply Supported Panel

Reference 33) is derived from linear analysis based on strain equations that were used by Donnell (34). Mode shapes for flat plates are assumed to be unchanged, and frequency behavior is influenced by inplane stress that is induced by lateral displacement. The frequency change of significance occurs in the manner shown by the frequency equation

$$\omega^2_{rs_{curved}} = \omega^2_{rs_{flat}} (1 + G_{rs} N^2)$$

Table I has been prepared to show the constants G_{rs} for a curved panel whose edges are clamped against edge rotation. The crowning, or curvature factor $N = (h_o/t)$ is the number of thicknesses that the center of the panel is displaced in its initially curved condition. In this instance, r denotes the number of half waves in the axial (flat) direction and s denotes the number of half waves in the circumferential (curved) direction.

Note that the modal designation m-n is flow orientated and that the designation r-s depends on the direction of curvature. In the same manner, the edge dimensions ℓ, w are stream flow oriented while ℓ', w' are oriented according to curvature. Figure 18 presents the thickness correction factor due to curvature as a function of length-to-width ratio. These results were obtained from four mode flutter studies using the analysis techniques described in Section IV.

TABLE I
Constants G_{rs} for a Clamped Curved Panel

Mode r-s	Mode shape	G_{rs}
1-1		$\dfrac{5.64}{5.144 + 3.115 (\ell'/w')^2 + 5.144 (\ell'/w')^4}$
1-2		0
1-3		$\dfrac{1.06}{5.144 + 25.015 (\ell'/w')^2 + 150.063 (\ell'/w')^4}$
1-4		0
2-1		$\dfrac{5.64}{39.063 + 11.626 (\ell'/w')^2 + 5.144 (\ell'/w')^4}$
3-1		$\dfrac{5.64}{150.063 + 25.015 (\ell'/w')^2 + 5.144 (\ell'/w')^4}$
4-1		$\dfrac{5.64}{410.063 + 43.393 (\ell'/w')^2 + 5.144 (\ell'/w')^4}$

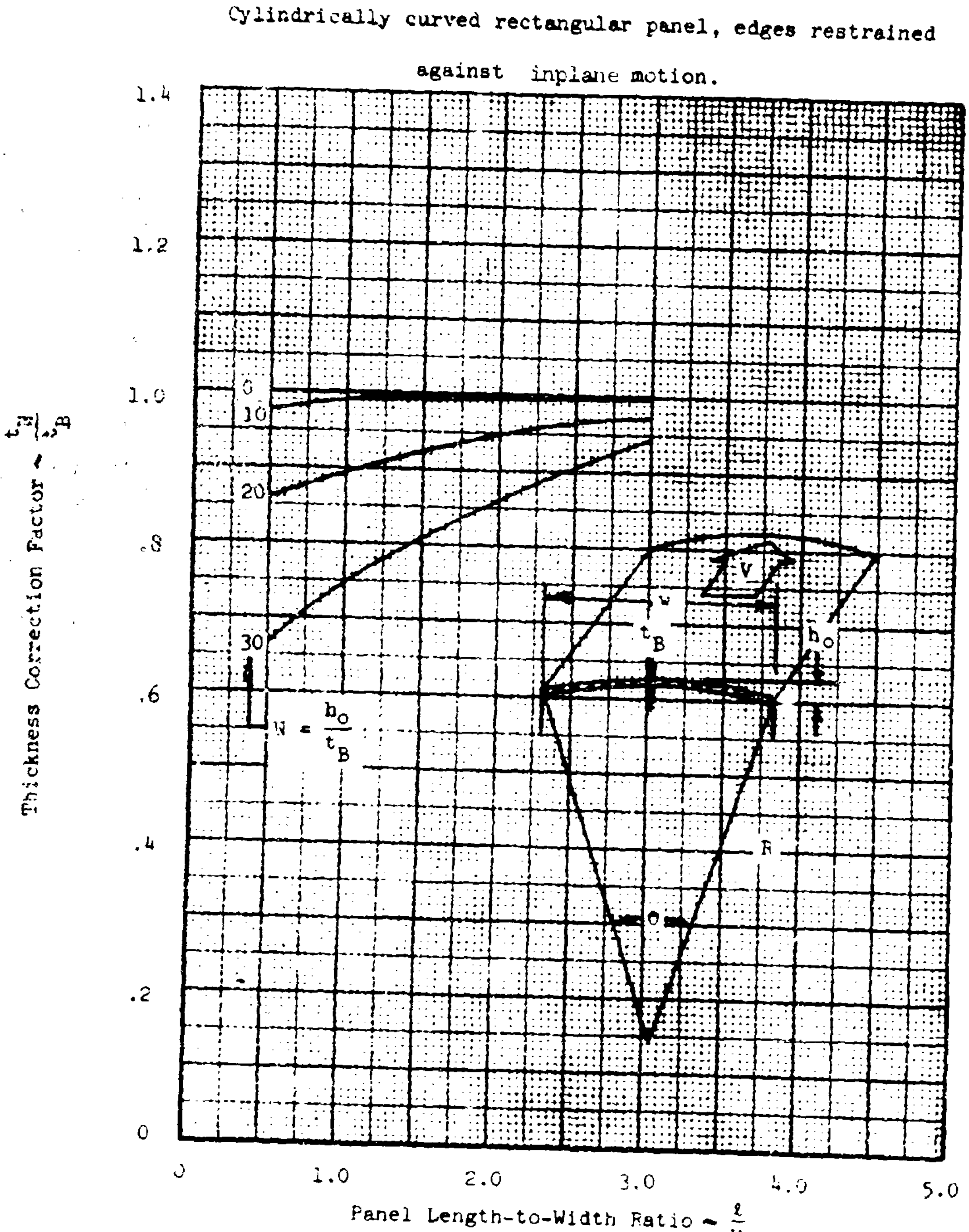


Figure 18 - Thickness Correction Factor for Curvature

8. Inplane Stress (N_x)

Inplane stress may result from differential temperature flightloads or manufacturing and installation procedures. The stress may be tensile or compressive; in comparison with an unstressed panel, the tensile stress raises flutter speed and the compressive stress lowers flutter speed.

The tension case has not been extensively studied because of the basic conservatism involved when a panel designed for zero stress is then subjected to tension. Experimental work reported in Reference 6 indicates that tension may be conveniently accounted for in design by using an effective value of the Young's modulus

$$E_{eff} = E\ (1 + \frac{N}{|N_{x_{cr}}|})$$

in which $N_{x_{cr}}$ denotes the longitudinal buckling stress with $N_y = 0$ (see Figure 19). If the effective value of Young's modulus is used in the panel flutter parameter, and if all factors other than t are held constant, then one obtains a thickness correction factor for tension stress

$$\frac{t_\sigma}{t_B} = (1 + \frac{N}{|N_{x_{cr}}|})^{-1/3}$$

The effects of compression stress are of greater concern to the designer, and at the same time are more difficult to assess. Flutter of flat panels under compressive stress is more difficult to analyze because

(a) edge support stiffness plays a more important role in determining panel dynamics when the panel is compressed than when it is under zero (or tension) stress, and

(b) the flutter instability of compressed panels is apparently influenced by static buckling (indicated by test) although theory indicates that the flutter boundary is determined by dynamic considerations alone.

Although the detrimental effects of elastic edge restraint stiffness is recognized (Reference 19) it is not feasible at present to formulate a criterion; additionally it is important to note that this problem is much more likely to occur with heavy, built-up panels than with single thickness skin panels.

The flutter speeds of compressed and buckled panels (studied extensively both analytically and experimentally) vary with both ℓ/w and bending edge conditions. The work that is reported in References 17 and 28 indicate that a compressed panel experiences its minimum flutter speed when the compressive load is near the still air buckling load. This apparent proximity of minimum flutter speed to the still air buckling load is contrary to the flat plate theory which indicates that the dynamic instability is not related

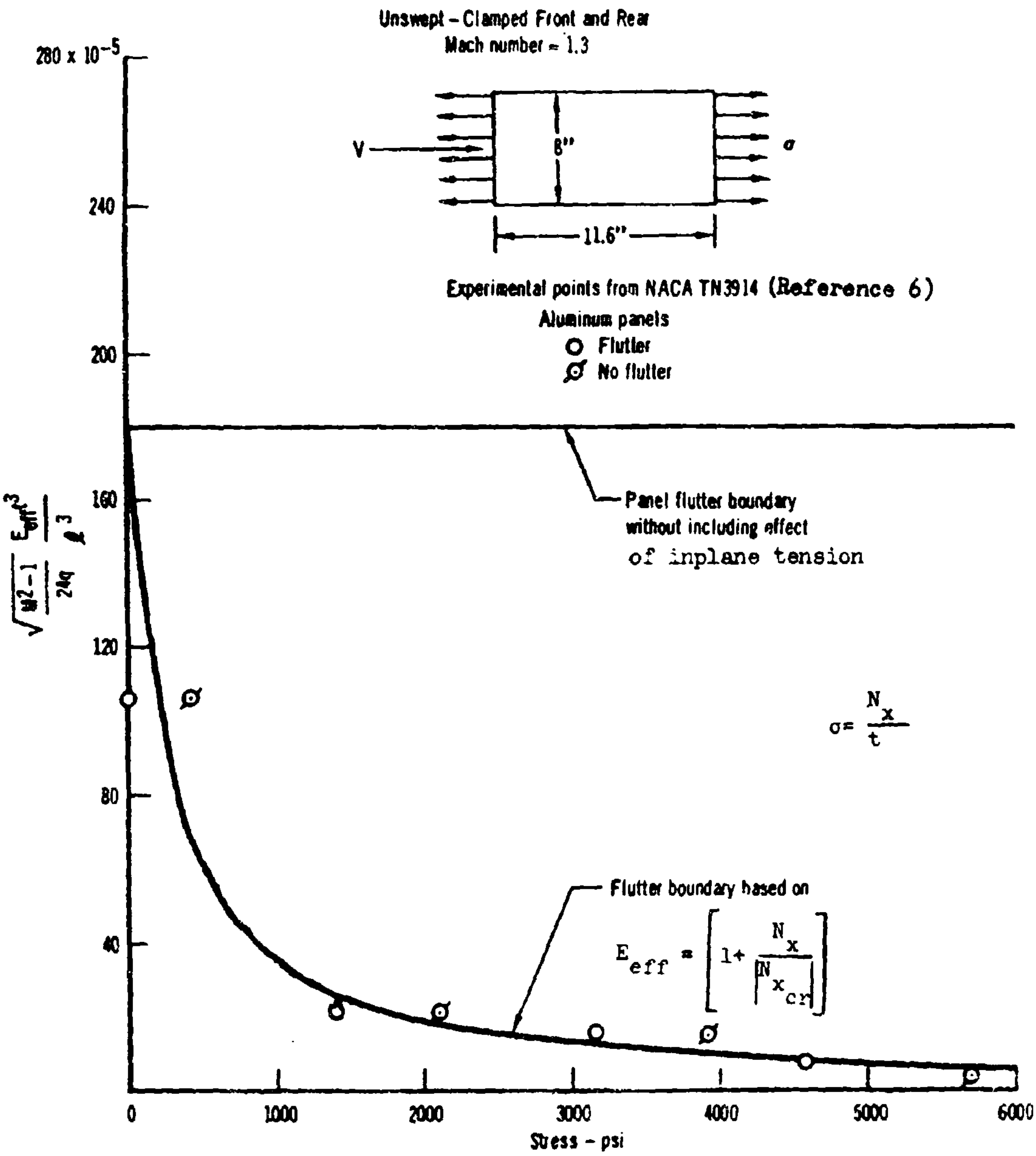


Figure 19 - Effects of Inplane Tension on the Flutter of Flat Panels

to the buckling condition. Theory also indicates that the lateral stress condition does not affect the flutter boundaries (see for example, Figures 13 and 15 of Reference 23). The theory, however, fails to account for the fact that a buckled panel exhibits dynamic behavior that cannot be taken into account using flat plate theory. The procedure that is proposed in this report consists of defining (theoretically) a critical streamwise load $N_{x_{cr}}$ on the basis of (a) buckling or (b) coalescence of still air frequencies of flutter critical modes. The value used depends on which of the conditions occurs at the lower value of compressive stress. This means, of course, that cross-stream load $N_y = \sigma_y t$ enters the panel design and the two conditions $N_y = 0$ and $N_y = N_x$ will be considered in this document.

The variation in the flutter boundaries at values of stress less than critical is not well defined experimentally although a substantial amount of data is available. The attempts to correlate experiment and theory for this report indicate that it is difficult to determine $N_{x_{cr}}$ for a real panel. The data correlations shown in Figures 20, 21, 22, and 23 compare an empirical-theoretical trend curve (solid) with data points that encompass a wide range of length-to-width ratios and stress levels. The solid line was adapted from theory given in Reference 23 and connects the zero stress level with the experimental maximum that was obtained at "buckling" in Reference 17. The present authors attribute the scatter to variation in N_x caused by the loss of panel stiffness due to exposure of flutter. The approach that has been selected for these criteria incorporates the thickness correction factor shown in Figure 24, as a function of $N_x/N_{x_{cr}}$ where $N_{x_{cr}}$ has been defined in the previous paragraph. The streamwise load ratio of Figure 24 has been adjusted to account for deviations of measured values of $N_{x_{cr}}$ as compared with theoretical values of $N_{x_{cr}}$. Measured data almost always indicate that the actual buckling loads are smaller than the theoretical buckling loads. Curves have been prepared to show the critical loads for clamped plates with $N_y = 0$ (see Figure 25) and $N_y = N_x$ (see Figure 26). Data for these curves were obtained from Reference 35. The curves of Figures 27 and 28 show the same type of information for simply supported panels. Note that $N_{x_{cr}}$ is determined by buckling for $\ell/w < 1.4$ and by frequency coalescence for $\ell/w > 1.5$ with $N_y = 0$ for simply supported panels (see Figure 27).

9. Buckling

Extending the discussion of compression stress that was begun in the preceding paragraph, it is recommended that a thickness correction factor of 2.0 be applied for buckling. The value 2.0 has been determined empirically (Reference 17) and should be used when the designer anticipates that the panel will be subjected to combined (N_x and N_y) loading that is capable of buckling the panel. Inherent in this assumption also is that no subsequent stress condition (post buckling) will cause a lower flutter speed.

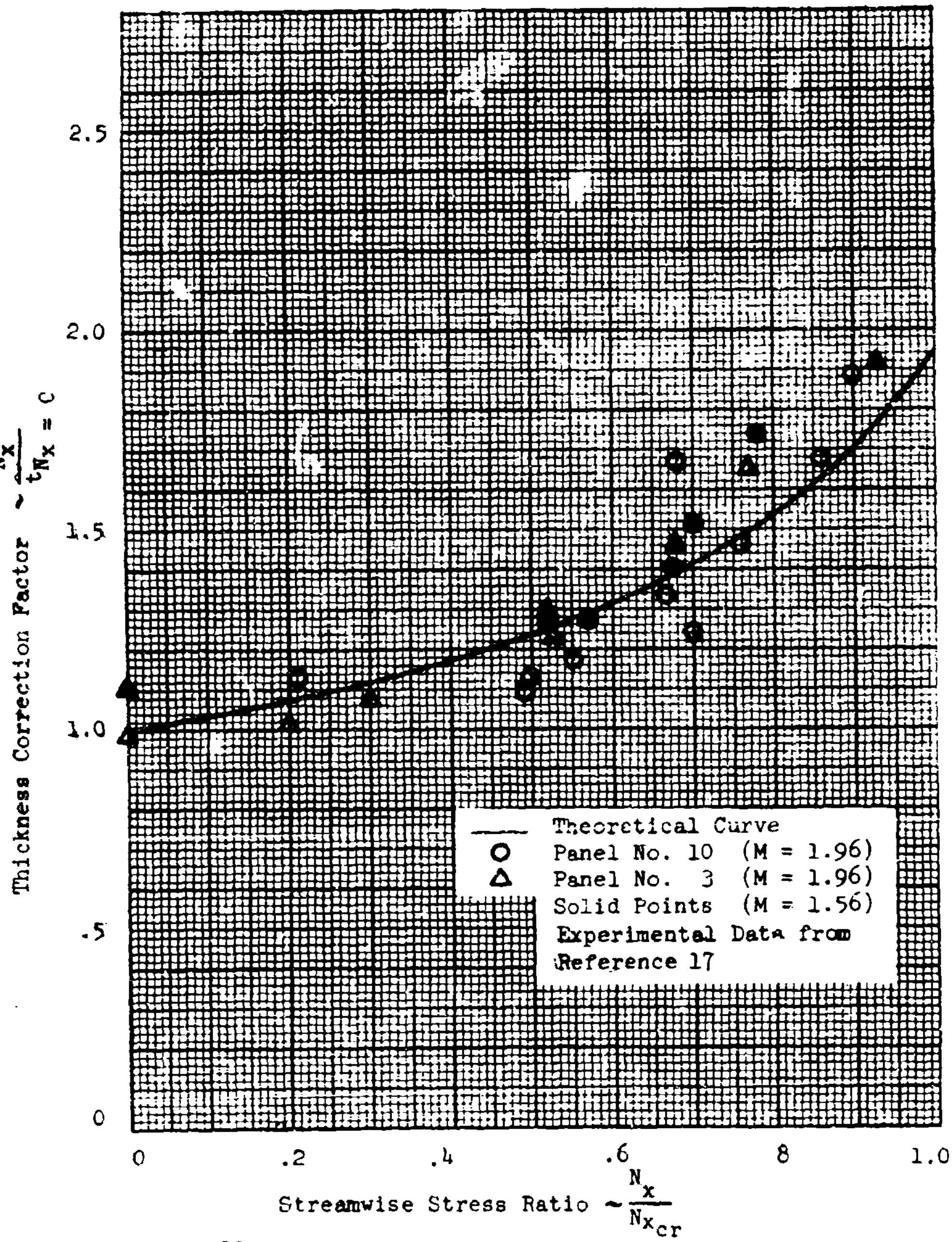


Figure 20 - Comparison Between Theoretical Correction Factor For Inplane Stress and Experimental Data for ($\frac{l}{w}$ = 1.0) (Unsupported Length)

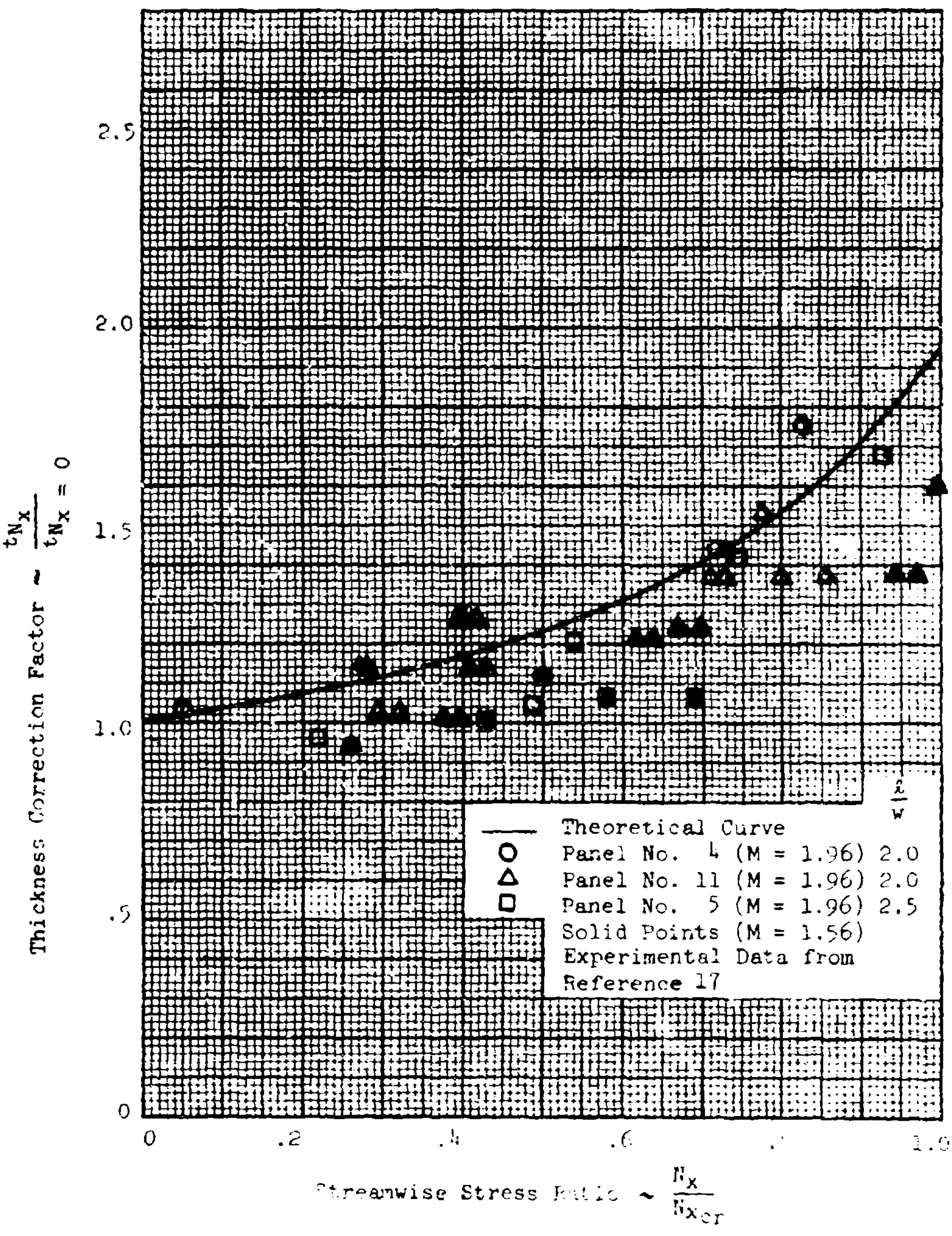


Figure 21 Comparison Between Theoretical Correction Factor For Inplane Stress and Experimental Data for ($\frac{\ell}{w}$ = 2.0 and 2.5) (Unsupported Length)

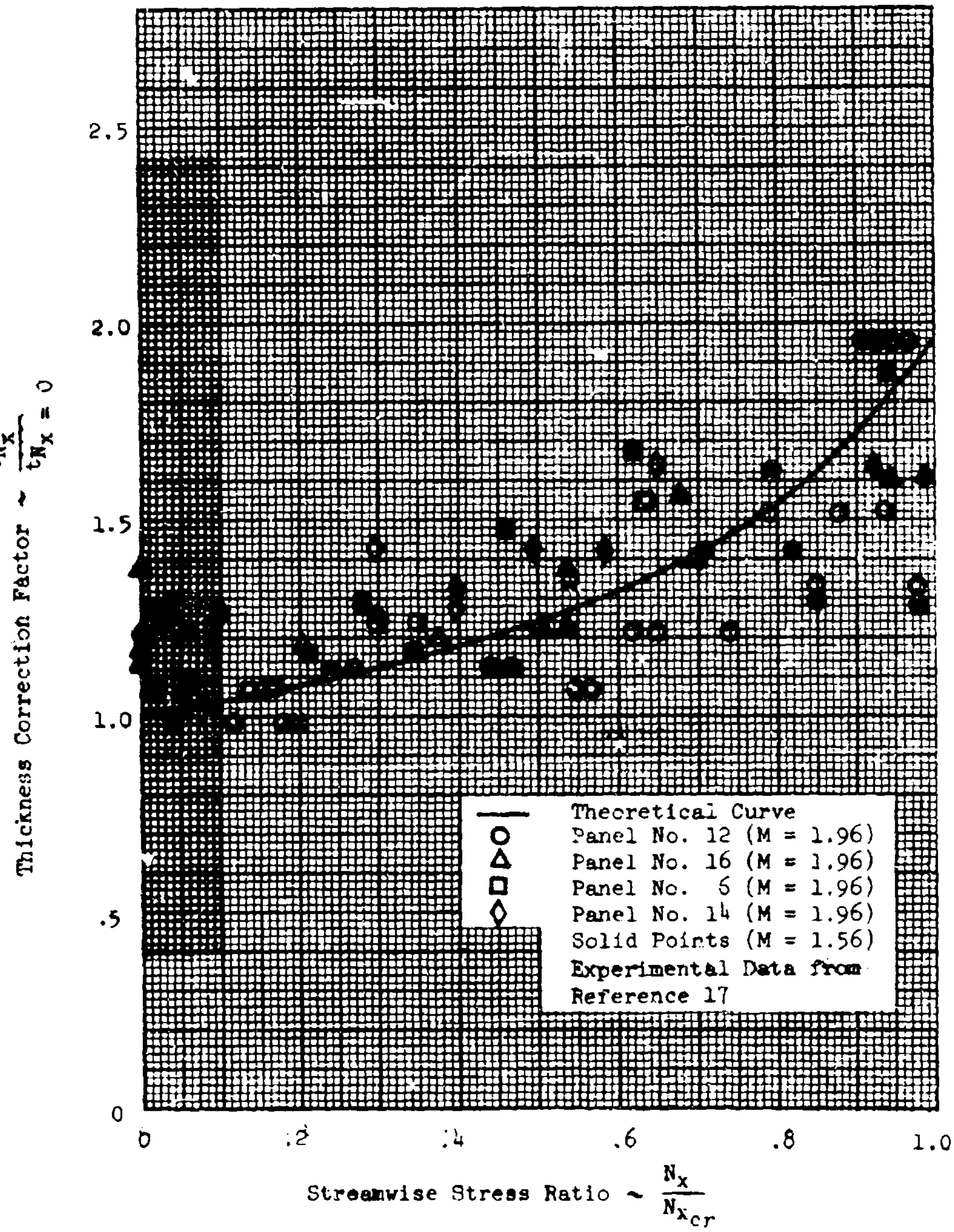


Figure 22 - Comparison Between Theoretical Correction Factor For Inplane Stress and Experimental Data for ($\frac{L}{W}$ = 3.0) (Unsupported Length)

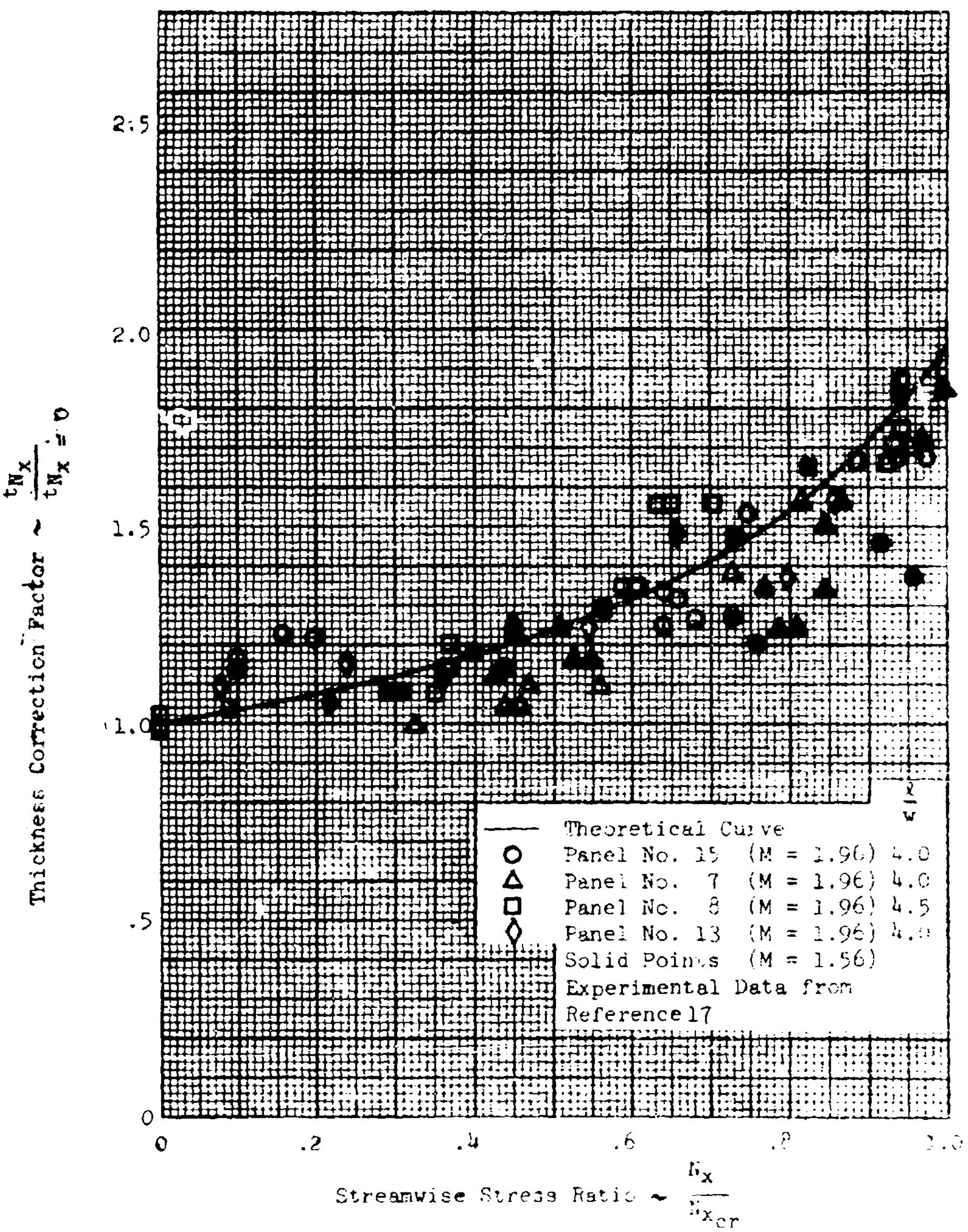


Figure 23- Comparison Between Theoretical Correction Factor For Inplane Stress and Experimental Data for $\frac{l}{w}$ = 4.0 and 4.5 (Unsupported Length)

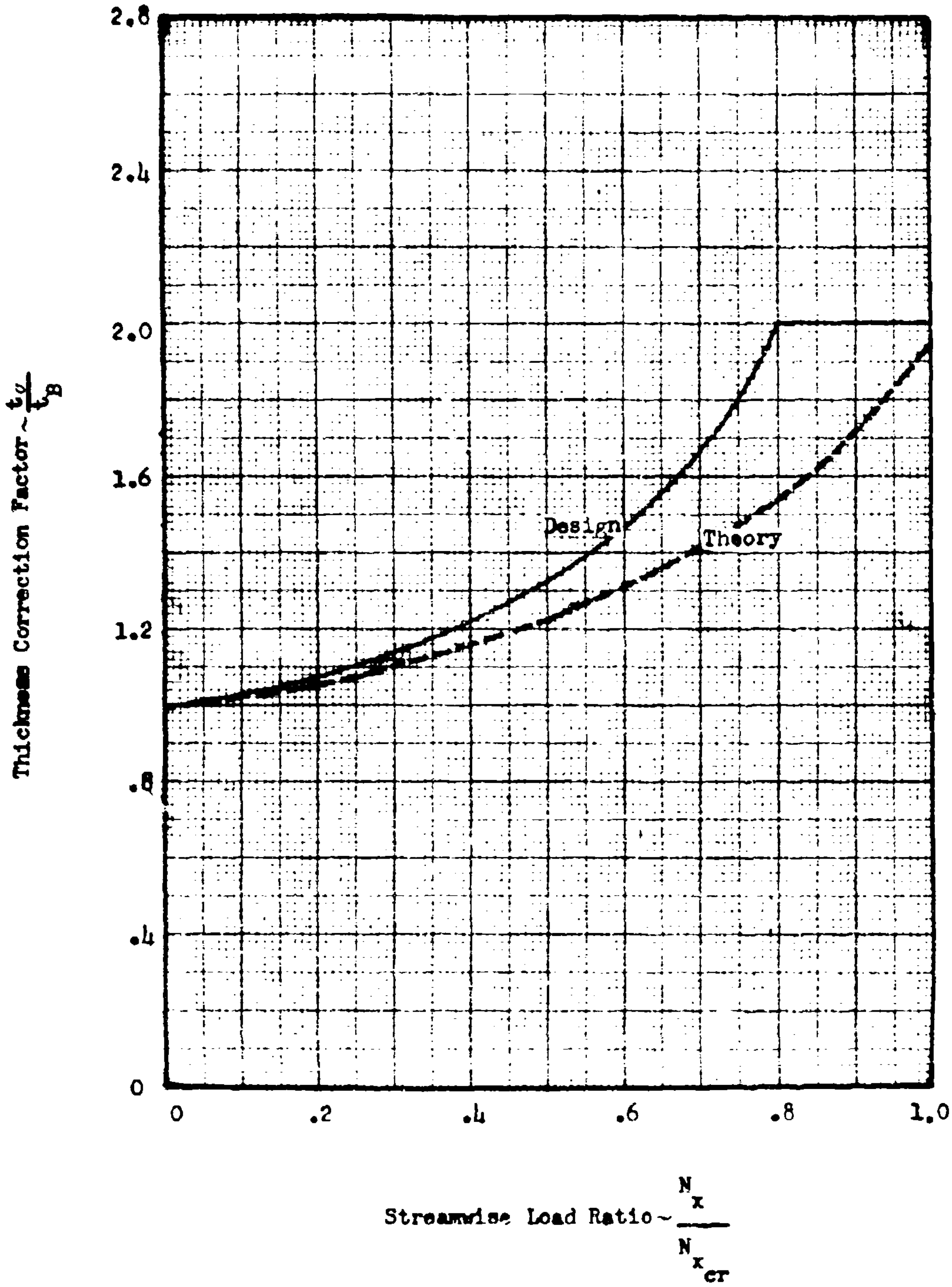


Figure 24- Thickness Correction Factor for Inplane Stress (Compression)

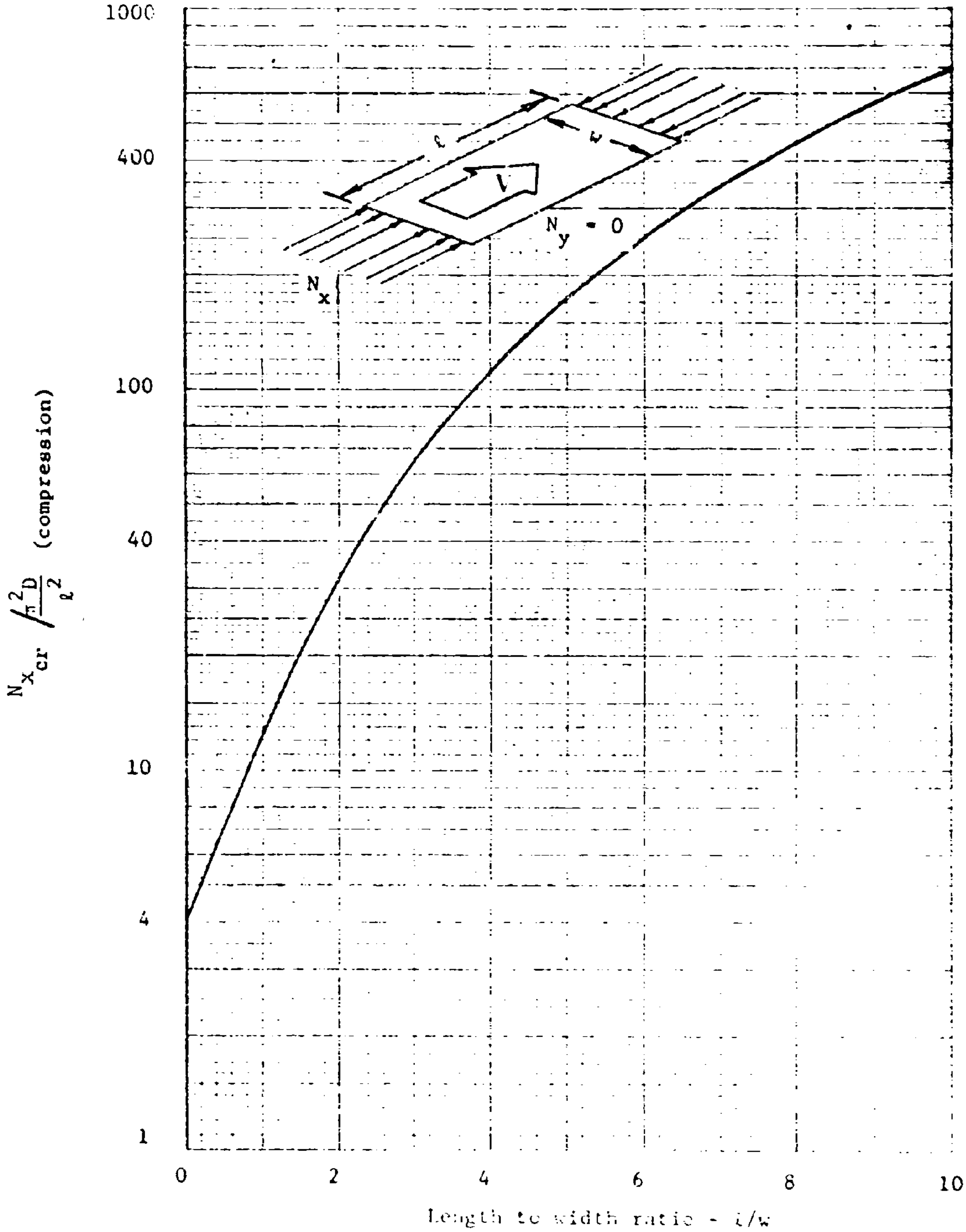


Figure [illegible] - Critical Inplane Compression Load for Clamped Panel, $N_y = 0$

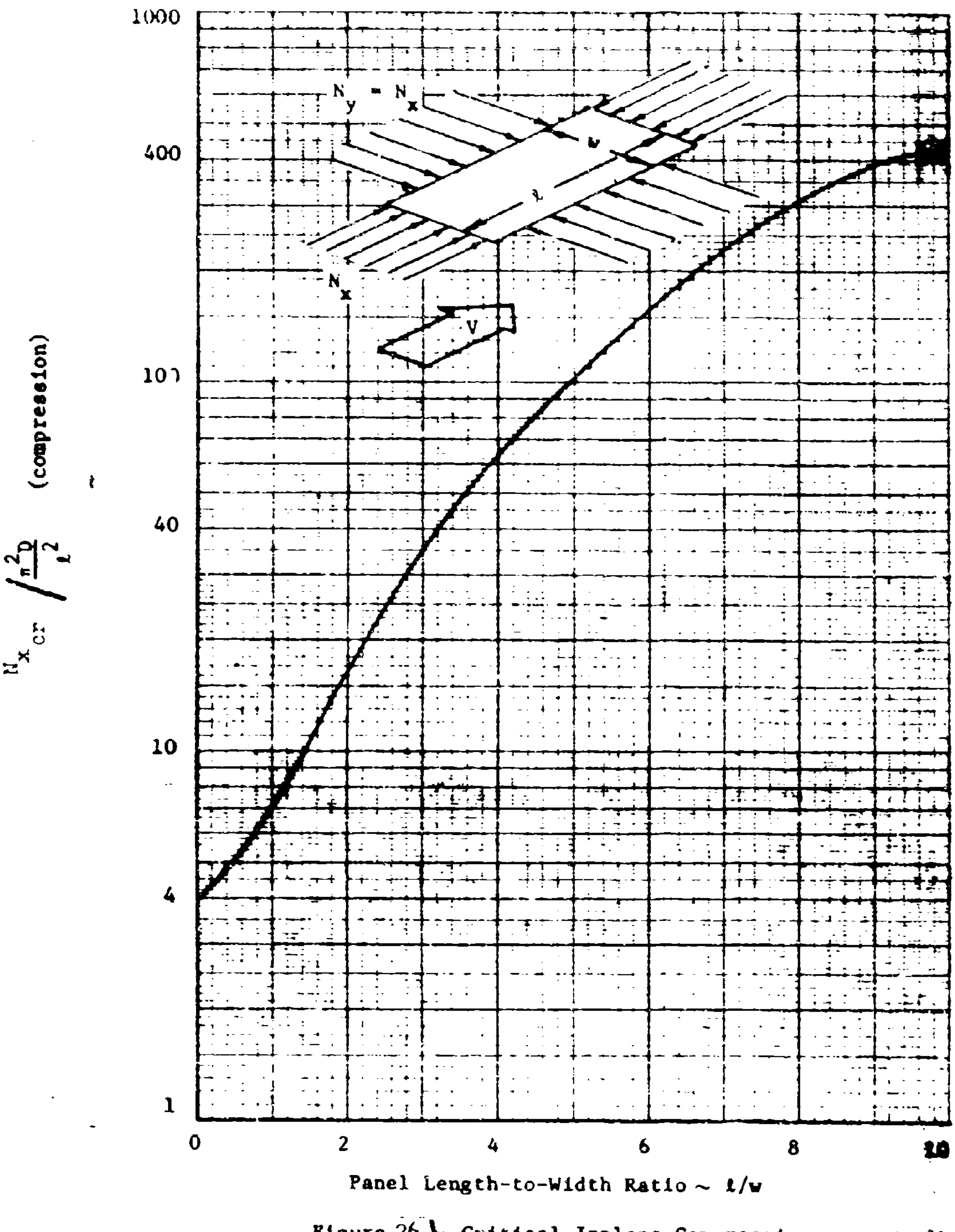


Figure 26 Critical Inplane Compression Load for Clamped Panel, $N_x = N_y$

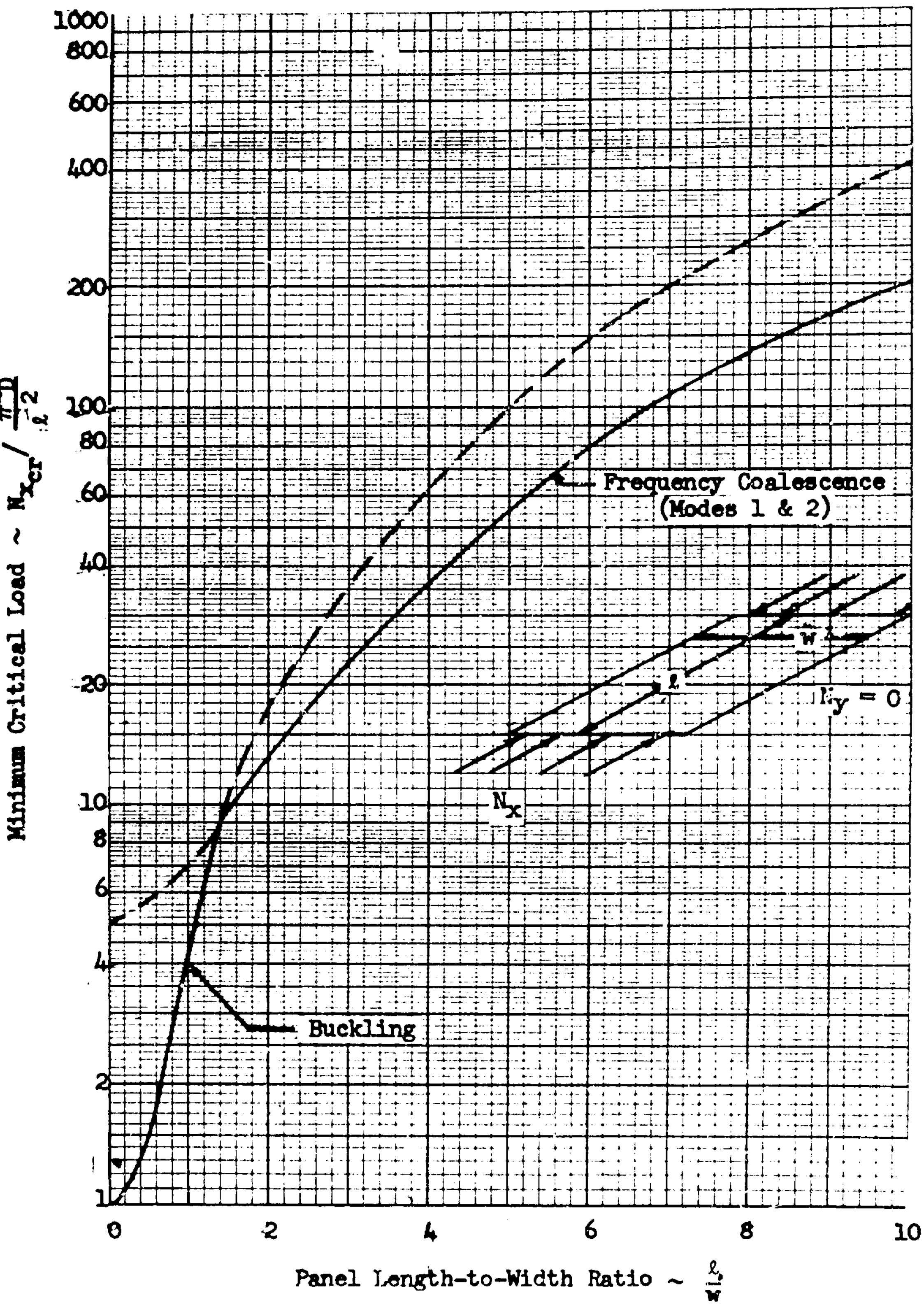


Figure 27 - Critical Inplane Load for Simply Supported Panel with $N_y = 0$

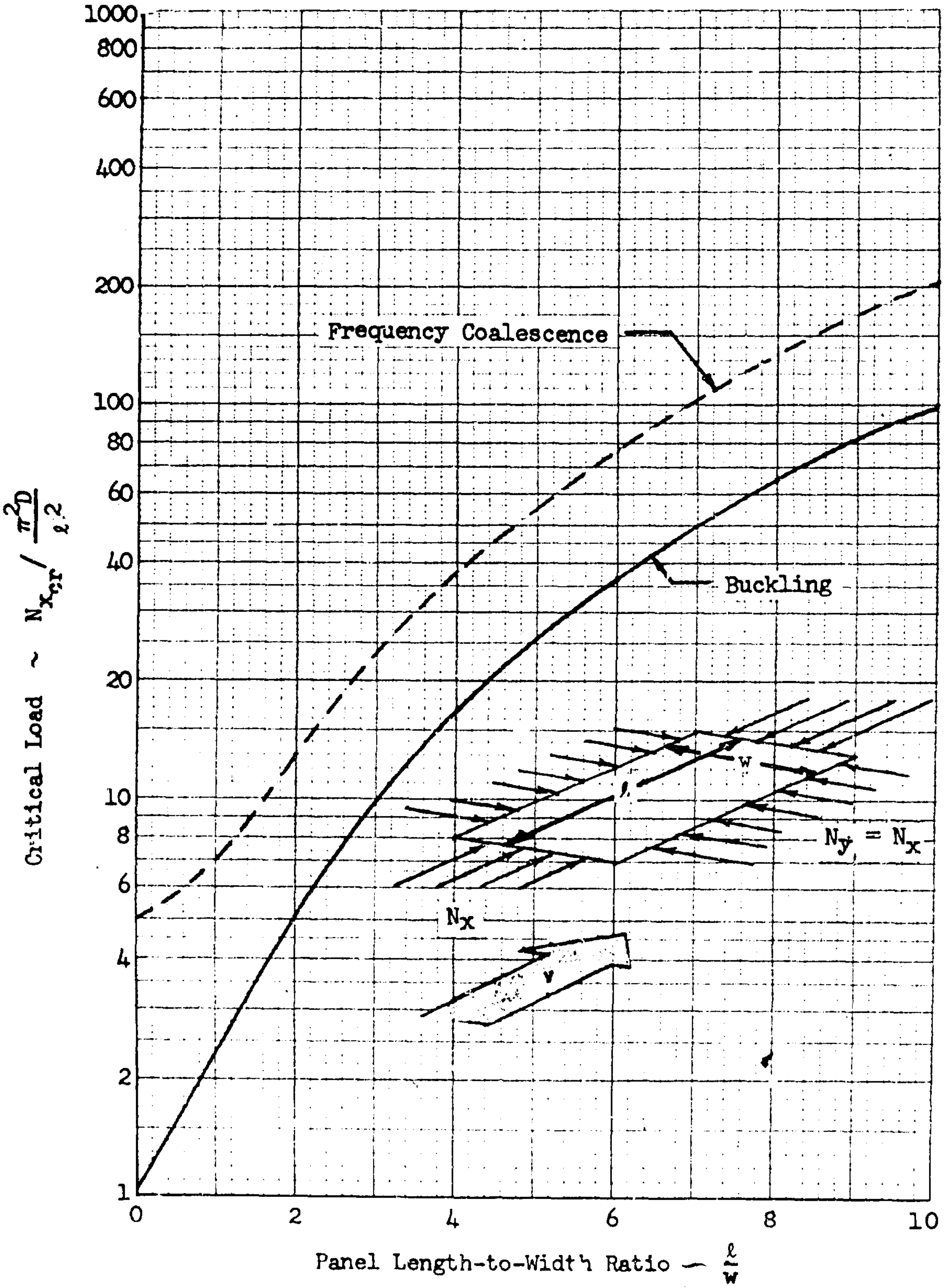


Figure 28 - Critical Inplane Load for Simply Supported Panel with $N_y = N_x$

10. Differential Temperature (ΔT)

Assume that the panel is under no inplane stress when it is at the same temperature as its supporting structure. If the panel temperature changes by an amount ΔT relative to its support, then the induced thermal stress is

$$\sigma = E\alpha_T \Delta T \qquad \text{or} \qquad N = (E\alpha_T \Delta T)t$$

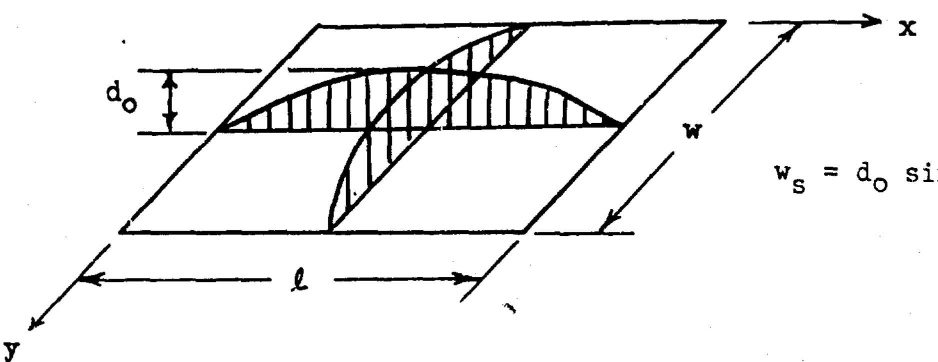
It is also assumed that the panel edges are restrained in-plane. A convenient reference quantity is ΔT_{cr}, which is the temperature change at which buckling would occur. The theoretical values of ΔT_{cr} for clamped and simply supported panels are shown in Figures 29 and 30 respectively for the case of $N_y = N_x$. The thickness correction factor will be obtained from Figure 31 by determining the resulting $\frac{N_x}{N_{x_{cr}}}$ for the panel, where $N_{x_{cr}}$ is taken as 80% of the critical buckling load or the frequency coalescence load, whichever is lower.

Temperature effects on inplane stress and buckling using experimental data from Reference 26 was investigated and the comparison between experiment and theory is presented in Figure 31. Reasonable trend correlation is obtained for the $\ell/w = 10$ panel.

11. Differential Pressure (Δp)

When a panel is subjected to supersonic flow, it is highly probable that different static pressures will exist on the upper and lower surfaces thereby creating a differential pressure Δp across the panel. The primary effect of Δp on the flutter characteristics of flat panels is to induce tensile stresses in the plane of the panel which result in increased stability.

An approximate solution technique was developed to determine the effect of differential pressure on the stability characteristics of hinged edge panels. The panel was assumed to deform, under the differential pressure, into the following shape.



$$w_s = d_o \sin \frac{2\pi x}{\ell} \sin \frac{2\pi y}{w}$$

which sets up static stresses in the midplane of the panel. By equating the work done by the differential pressure to the bending energy of the panel plus the membrane energy associated with the inplane tension loads, a solution was obtained for the tension loads in the x and y directions. An approximate solution of the panel flutter problem was then obtained by using Lagrange's equation in conjunction with two-dimensional static aerodynamics. The derivation of the

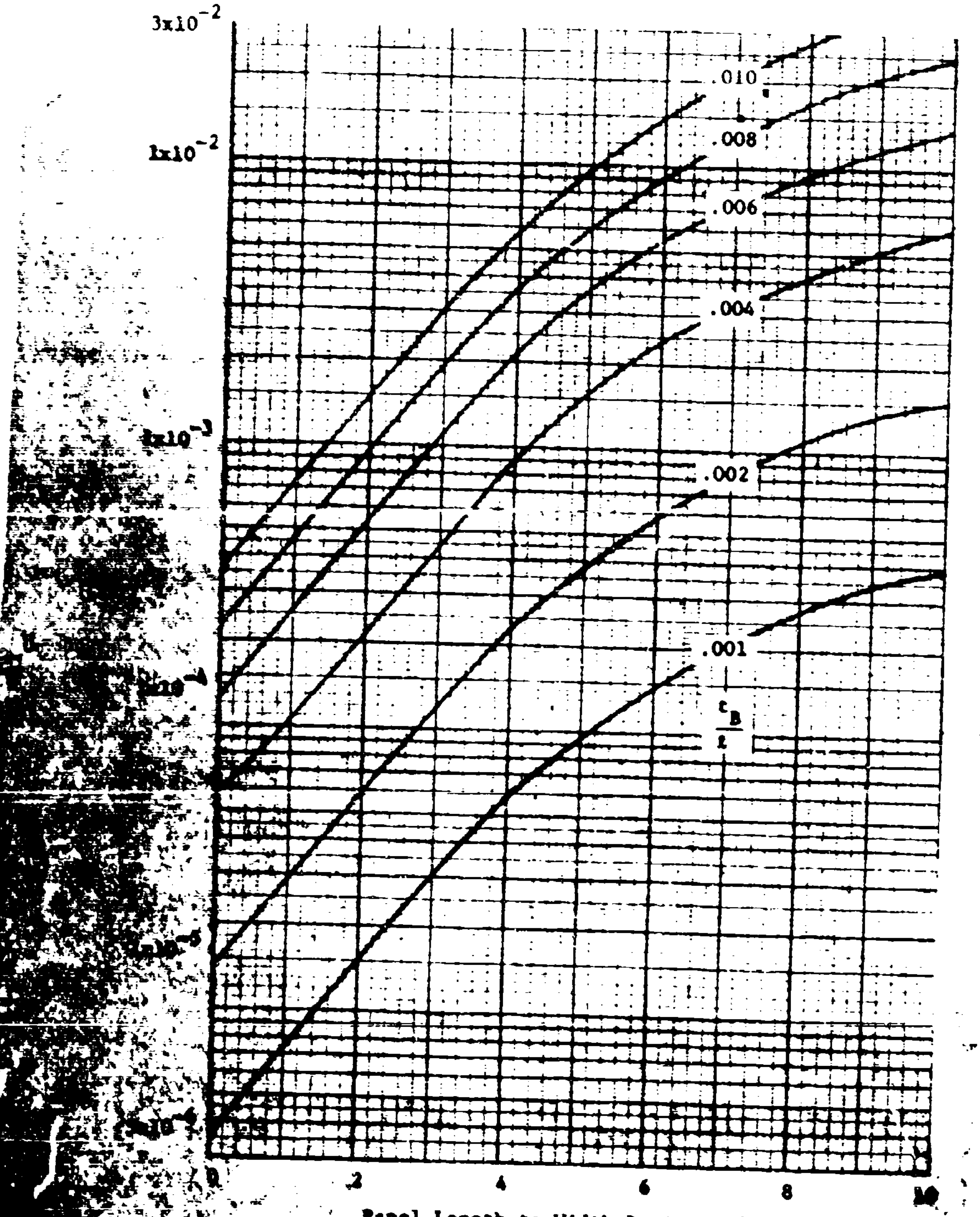


Figure 29 - Critical Differential Temperature of Clamped Panel with Restrained Edges

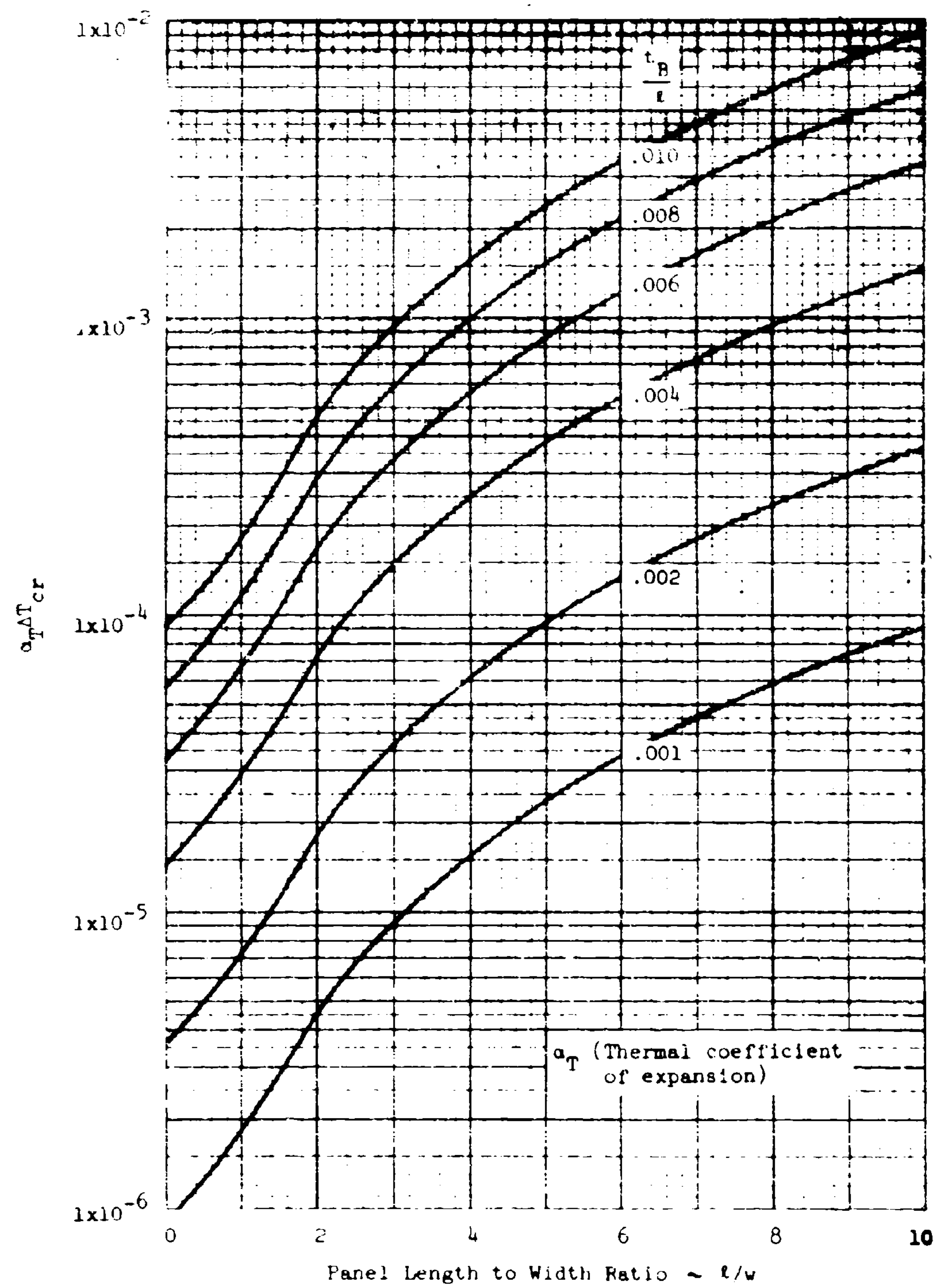


Figure 30 - Critical Differential Temperature of Simply Supported Rectangular Panel with Restrained Edges

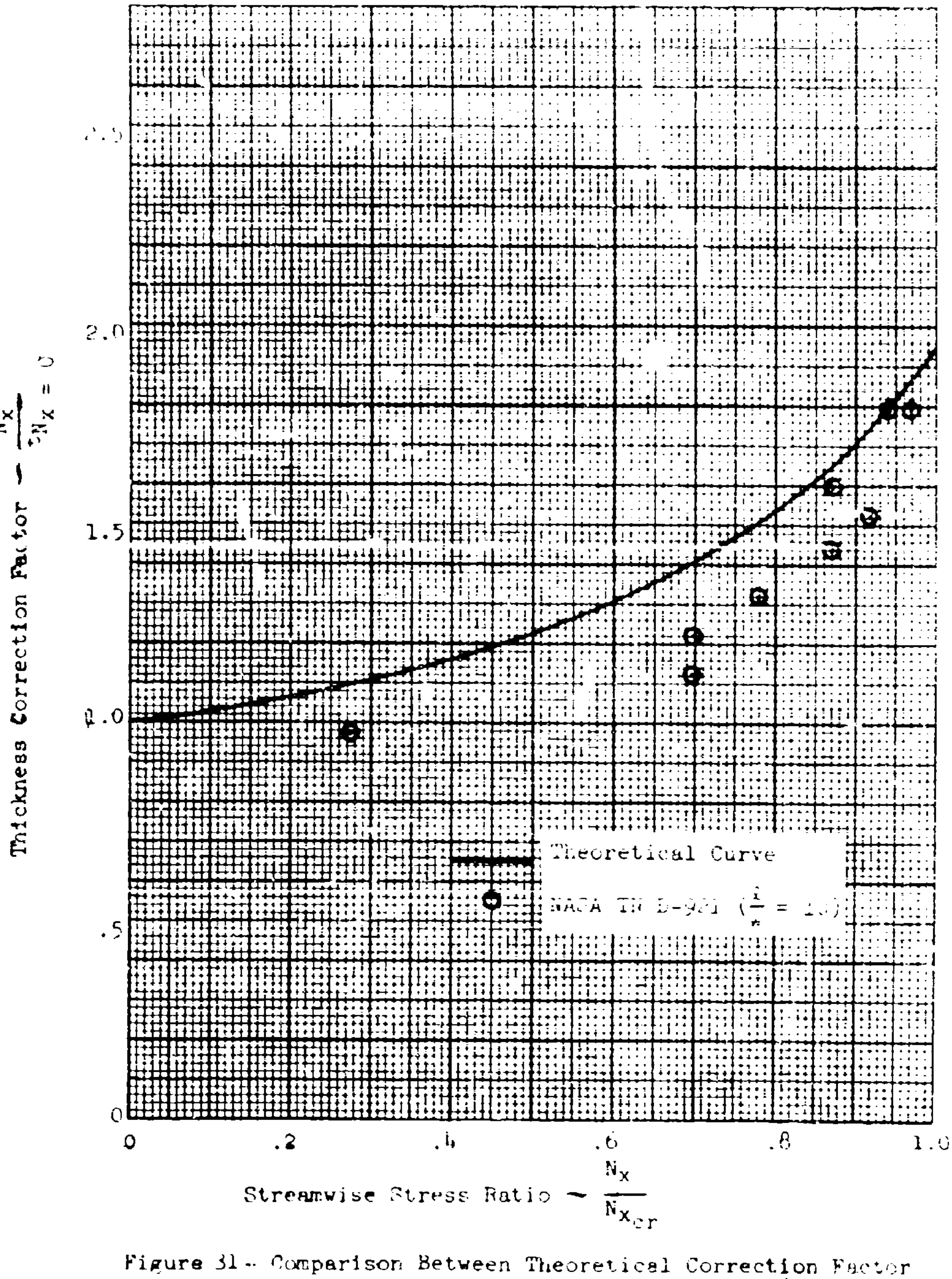


Figure 31.- Comparison Between Theoretical Correction Factor For Inplane Stress and Experimental Data for ($\frac{l}{w}$ = 10)

tensile stresses in the plane of the panel and the flutter stability equations of a panel, acted upon by differential pressure, are presented in Appendix B.

Results of the analyses are shown in Figures 32 and 33. Figure 32 shows the relationship between the nondimensional pressure parameter $\Delta p/E \cdot (t/\ell)^4$ and the crown height parameter d_o/t, while Figure 33 shows the thickness correction factor, $t_{\Delta p}/t_B$, as a function of ℓ/w with parametric variation of d_o/t.

The results shown in Figure 33 were obtained using four stream modes and one cross-stream mode for length-to-width ratios of one half, one and two while eight stream modes and one cross-stream mode were used for the length-to-width ratios of three and four.

12. Cavity Effect

A cavity is defined here as an enclosure behind a panel in which air is contained. The effect of the cavity on panel vibrations, as discussed in Reference 16, is a problem in aeroelasticity; the principal effect is that of an aerodynamic spring acting on the fundamental mode and other modes whose mode shape deformations tend to alter the cavity volume. Virtual mass effects, due to movement of the constrained air during vibration, are negligible for panel and cavity sizes of practical interest. Analy , however, show that the aerodynamic spring effect is dependent on a nondimen nal expression

$$\lambda_c \frac{\ell}{d}$$

where

$$\lambda_c \frac{\ell}{d} = \rho \frac{a_o^2 \, \ell}{D \, d} = 1.4 \frac{p_{cav} \, \ell}{(D/\ell^3) d}$$

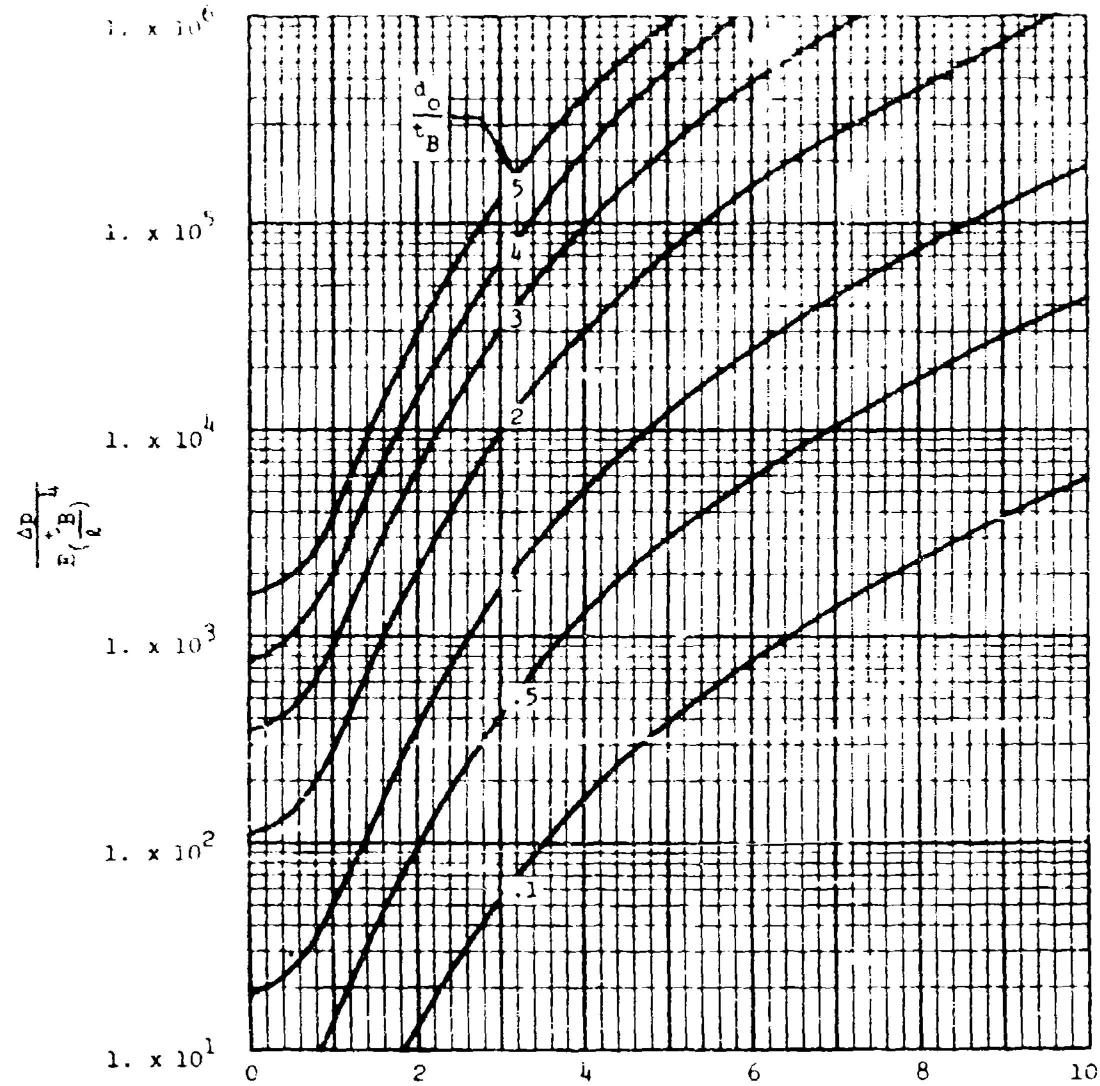
A one-term approximation for the fundamental mode (Reference 16) is

$$K_{11}^2 = K_{11_{vac}}^2 + 4/9 \, \lambda_c \frac{\ell}{d}$$

which by substitution becomes

$$\rho_p \frac{t\omega^2 \ell^4}{D} = \rho_p \frac{t\omega_{vac}^2 \ell^4}{D} + .62 \frac{p_{cav} \, \ell}{(D/\ell^3) d}$$

Flutter boundaries were obtained for values of this parameter as large as 9000 in a four-mode study with ℓ/w varying from 0 to 3. The results of the study, expressed as a function of the thickness ratio t_{cav}/t_B is shown in Figure 34. The flutter results are expected to be somewhat conservative as a result of using the one-term approximation.



Panel Length-to-Width Ratio ~ $\frac{\ell}{w}$

Figure 32 - Relationship Between $\frac{d_o}{t_B}$, Δp, and Panel Geometric-Physical Characteristics

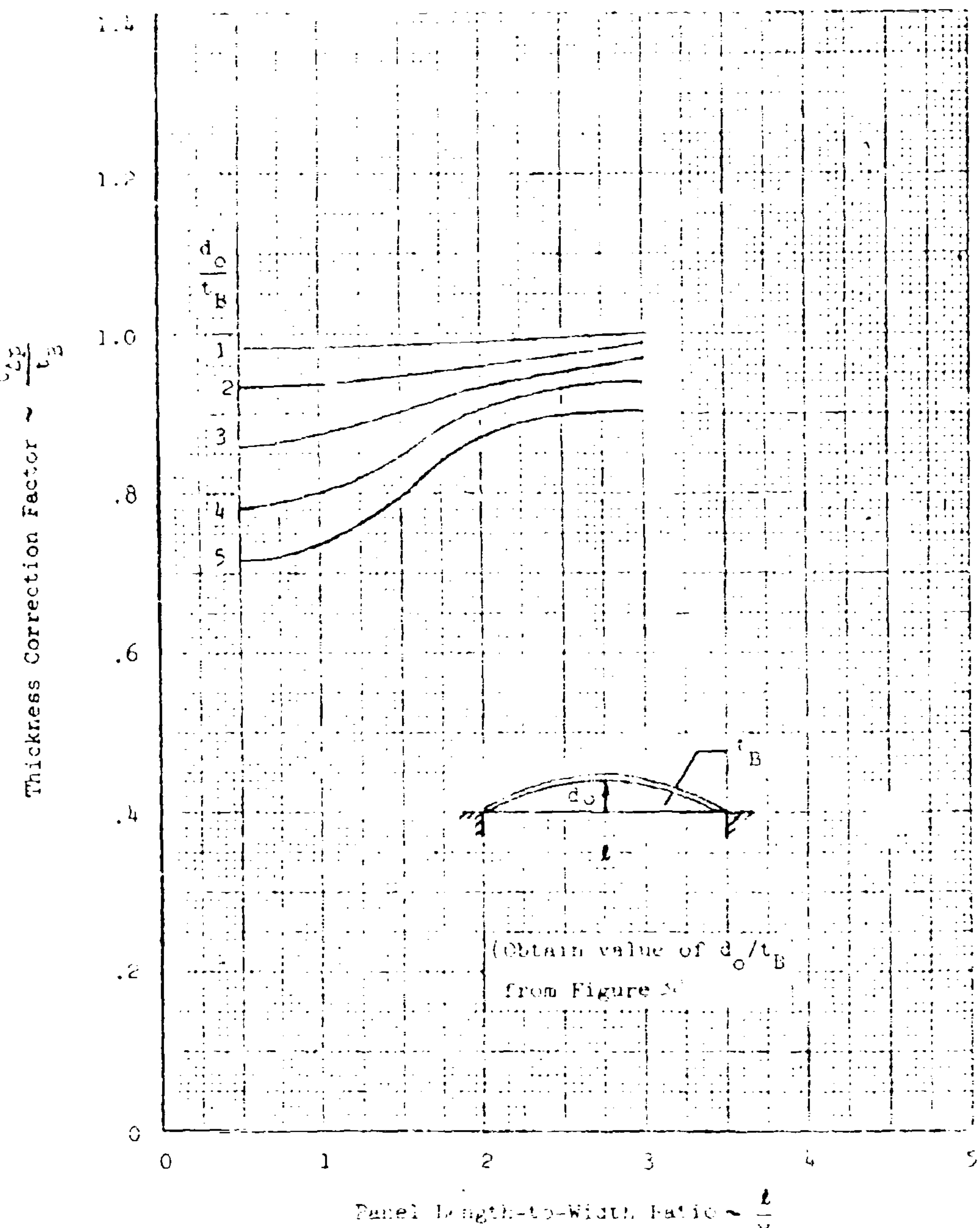


Figure 33 - Thickness Correction Factor for Differential Pressure

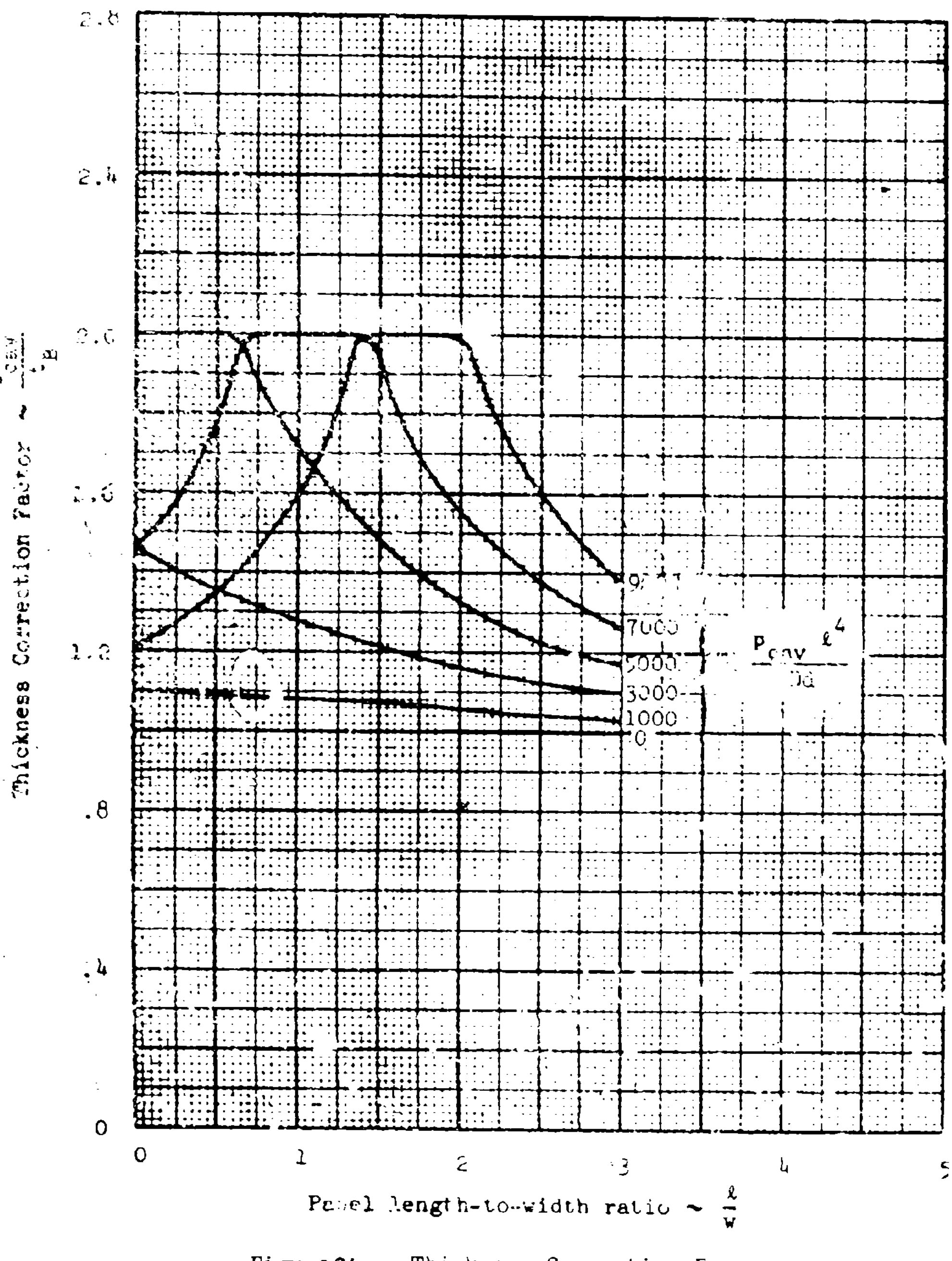


Figure 34. - Thickness Correction For an Enclosed Cavity

13. Orthotropicity

An orthotropic panel is defined here as one having unequal bending stiffness in two orthogonal directions. The use of such configurations results in considerable increases in bending stiffness in one direction over single thickness panels. In the most general cases, the orthotropicity results from corrugation backing applied to a flat face sheet; it may also result from stiffeners or beading. Figure 35 illustrates these common examples of orthotropicity.

The differential equation for an orthotropic panel is well known. It must be recognized, however, that edge support conditions become very important, especially in the case of built-up panels so that flutter prediction of orthotropic panels has proven difficult. An extensive series of tests was conducted by the Boeing Company as discussed in References 36 and 37. It is believed (References 18 and 19) that the major difficulty lies in dynamic analysis of the composite structure. Still air frequencies must be known very accurately before a flutter solution should be attempted.

14. Damping

System damping, whether due to aerodynamic or structural origins, dissipates energy that might otherwise tend to worsen an aeroelastic instability. However, damping may also induce phase angles between flutter critical modes, thus causing coalescence to occur at significantly lower airspeeds than for an undamped panel. Flutter tests of panels that were treated with damping compound, as reported in Reference 28, indicated

(a) that the treatment was stabilizing, but

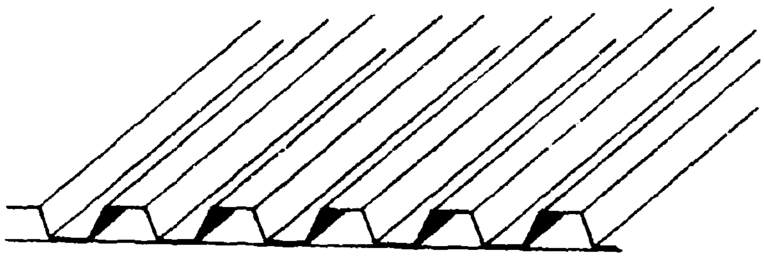
(b) the gain in flutter boundary could have been accomplished by using a thicker panel with a total weight significantly less than the weight of the original panel with the damping compound.

The test was restricted in overall scope (a single Mach number, two values of ℓ/w) and results are not sufficiently comprehensive to provide a basis for design criteria. An uncertainty that accompanies the uniform application of mass to a panel is the lowering of still air natural frequencies. This condition would likely result in diminished flutter speeds which might offset the increased stability offered by the damping.

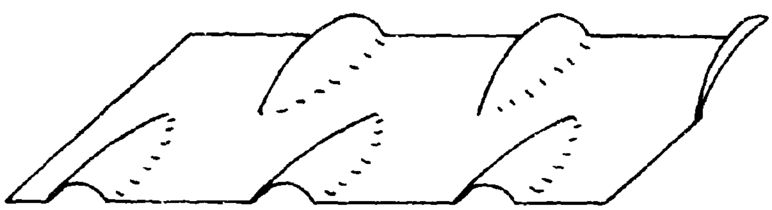
The following approach is recommended at the present time:

(a) Do not incorporate damping in initial panel design.

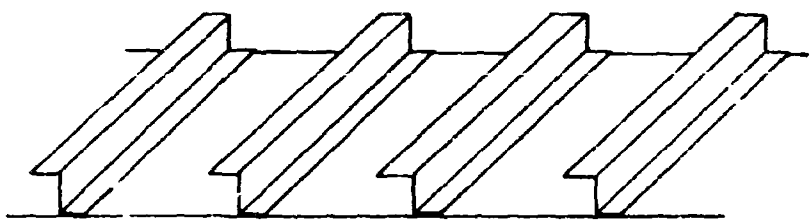
(b) Consider the application of damping compound if panel flutter is encountered in flight only if considerable weight increase can be tolerated, and other remedial measures are less feasible.



(a) Corrugation Stiffened



(b) Beaded



(c) Stringer Stiffened

Figure 35 - Examples of Orthotropic Panels

15. Boundary Layer

There has been disagreement in the past concerning the effect of boundary layer on panel stability. Experimental work is currently being done at the NASA Ames Aeronautical Laboratory to empirically determine boundary layer effects; preliminary results indicate that a boundary layer is stabilizing and the greatest effect on panel flutter occurs at M = 1.2. However, sufficient data are not available at present to define a criterion for panel design.

SECTION IV

DESIGN APPROACH

The poor correlation between theory and experiment has not only caused an unwarranted loss of confidence in analysis, but has also increased the burden of the designer who is responsible for designing panels against flutter. Extensive literature surveys and personal contacts were made during the course of this study; it was concluded that "anomalous" panel behavior during tests has been largely due to structural causes that were either ignored or unsuspected. This is not to say that all the theories are flawless; however, it should be possible to obtain much better correlations by making sure (1) that the physical condition of the panel under test is accurately known, and (2) that the structural analysis accounts for all aspects of the physical conditions that influence dynamic behavior. This document proposes a "mix" of available theory and experiment by combining good experimental data with theoretical interpolation or extrapolation into areas where such data are lacking.

1. Philosophy

The philosophy that has prevailed during the preparation of these criteria is summarized as follows:

(a) The onset of panel flutter is an aeroelastic instability that lends itself to analysis by linear methods; it is not unlike the flutter of lifting surfaces; therefore the established methods of analysis can be applied to panels in many of the areas of investigation (i.e., where modal methods are applicable). Furthermore, different mechanisms cause panel flutter in the supersonic (frequency coalescence) and transonic (negative damping) flow regimes, but the transition between them is smooth. Experimental trends can be used to extrapolate from the supersonic regime (where the instability is more amenable to analysis) to the transonic regime (where theoretical analyses have yielded erratic results).

(b) The unresolved problem areas are predominantly structural; hence the most fruitful areas lie in improving the mathematical descriptions of panels (and their supporting structures) to more accurately predict dynamic behavior.

The flutter boundaries of panels can only be determined if their still air dynamics can be predicted or measured. The importance of the panel spectral characteristics cannot be overemphasized. The panel flutter analyst must keep in mind that the best estimates of stability boundaries depend in large measure on accurate knowledge of panel dynamics.

(c) "Exact" solutions to panel flutter analyses, as described in References (10) and (23), provide solutions of certain classes of problems (such as very large ℓ/w with relative ease), and may be

valuable in establishing trends; modal solutions (Galerkin, Rayleigh-Ritz) are preferred for most analytical needs, however, because of the flexibility that is offered by the use of measured, or carefully calculated, modal frequencies. This latter type of analysis facilitates the use of vibration test data and correlation between theory and experiment. It must be recognized that each analytical approach has its advantages and disadvantages and should be used with discretion.

The experimental-theoretical approach to panel design that has been chosen for this set of criteria is relatively simple in concept. Since one of the more disturbing discrepancies between theory and experiment has been in establishing actual flutter speeds by theoretical means, this set of criteria offers a flat panel design that is based on experimental data; the thickness is modified by empirical-theoretical means to account for the physical parameters that have caused most of the problems in design.

The panel designer's first task is to accumulate the data, both aerodynamic and physical, that may influence panel design. The following procedure is followed after design inputs have been obtained.

(a) Establish a baseline panel design that is based on ℓ/w. The baseline panel is flat, unstressed, unswept, uniform and has all edges clamped. The baseline design curve is shown in Figure 36 as a plot of Φ_B versus ℓ/w. In the range $\ell/w = 1.0$ to 4.5, the curve is obtained from Reference 17. Tests described in this reference were formulated with full knowledge of prior test difficulties; they are believed to offer the best data available for use as criteria. The remainder of the curve is faired in by using theoretical trends obtained from Reference 23. Given ℓ/w, the designer determines Φ_B. (In addition to the baseline curve and data points from Reference 17, Figure 36 also presents an envelope curve from Reference 26. The curves agree well in the range $2 < \ell/w < 5$; discrepancies between the two curves outside this range are due mainly to the fact that the TN D-451 curve envelopes data from different panels under different flow conditions and hence would be expected to cause excessive overdesign for some applications. The baseline curve is extrapolated outside the range of LWP-177 data by theoretical means; further experimental investigations are needed in these areas.)

(b) Consider the baseline panel flutter parameter Φ_B (which is now known) in the form

$$\Phi_B = \left(\frac{f(M)}{q}\right)^{1/3} E^{1/3} \left(\frac{t_L}{\ell}\right) .$$

which separates the parameters into aerodynamic and structural parts. This step is primarily concerned with determining the aerodynamic part $\frac{f(M)}{q}$ of the baseline parameter. The quantity $f(M)$ replaces $\beta (= \sqrt{M^2-1})$ in the usual formulation of the panel flutter parameter. In essence, $f(M)$ accounts for the effect of Mach number on flutter speed. This variation has been predicted analytically by the use

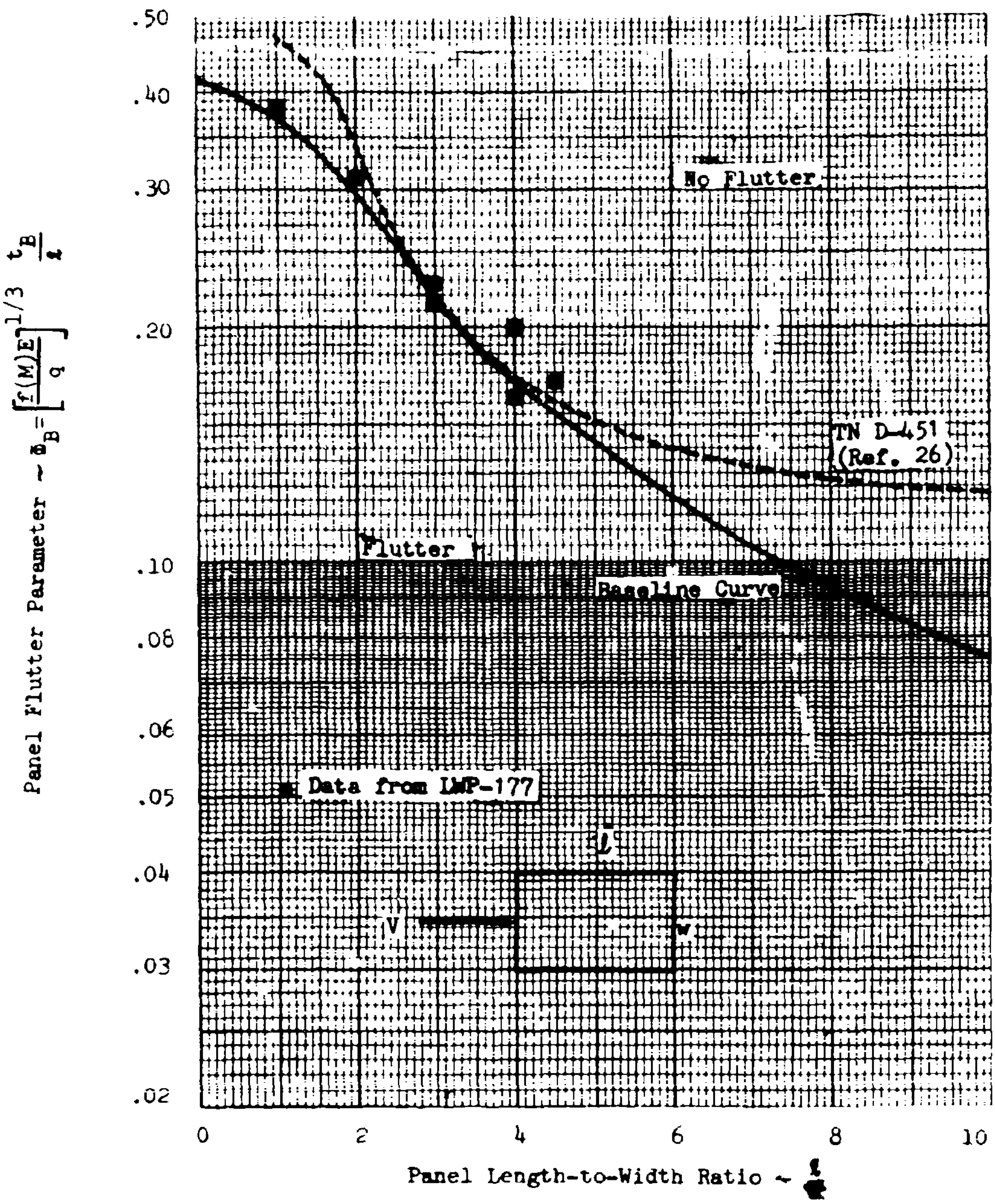


Figure 36- "Baseline" Design Curve

of three-dimensional, potential flow, linearized aerodynamic theory. The effects of Mach number on flutter speeds for panels of $\ell/w = 1/2$ and $\ell/w = 2$ are shown in Figures 4 and 5 . The data are normalized to values at M = 2 and lines of β are shown for comparison. The theoretical data shown on the plots do not offer sufficient information to formulate a criterion; therefore, envelopes were drawn to encompass the experimental data. (The differences in the envelope levels between Figure 4 and Figure 5 indicate an aspect ratio dependence at low Mach number.) The envelope of the data for $\ell/w = 1/2$ is proposed as the function of f(M). (A degree of conservatism is introduced at larger ℓ/w, but existing data are insufficient to formulate a criterion.) The designer is required to obtain the minimum value of $\frac{f(M)}{q}$ which is seen to relate to the maximum value of $E^{1/3}\,\frac{t_B}{\ell}$ through $\left[E^{1/3}\,\frac{t_B}{\ell}\right]_{max} = \Phi_B \Big/ \left[\frac{f(M)}{q}\right]_{min.}$

One procedure that can be used is to obtain the M-q relationship for the anticipated flight envelope. From M determine f(M) from Figure 6 then divide by the value of q. Compute a sufficient number of values to determine the minimum (critical) value. This value is used to enter the abscissa of Figure 37 . In order to expedite the procedure, however, the plot of Figure 38 can be used directly by the designer if he has a flight envelope in terms of Mach number and altitude. The necessary conversion has been made so that the flight envelope point on the graph is the maximum (critical) value of $\frac{q}{f(M)}$. Figures 37 and 38 have been consolidated through their common abscissa in Figure 39. A sample trajectory is shown on Part B of the curve for a panel of $\ell/w = 3$. The intersection of the two lines obtained from $\left[\frac{q}{f(M)}\right]_{max} = 4400$ and $\ell/w = 3$ gives the required value $E\left(\frac{t_B}{\ell}\right)^3 = 0.31$. With E and ℓ specified, then

$$t_B = \ell\sqrt[3]{\frac{0.31}{E}}$$

(c) Beyond the baseline case, the set of criteria must account for changes in panel thickness that are required to accommodate changes in flutter speed that are caused by the physical parameters. The following ground rules have been followed in this step.

(1) Experimental data are used where it is sufficiently comprehensive to establish trends.

(2) Theory is used to supplement experiment when the data are insufficient to fully establish trends.

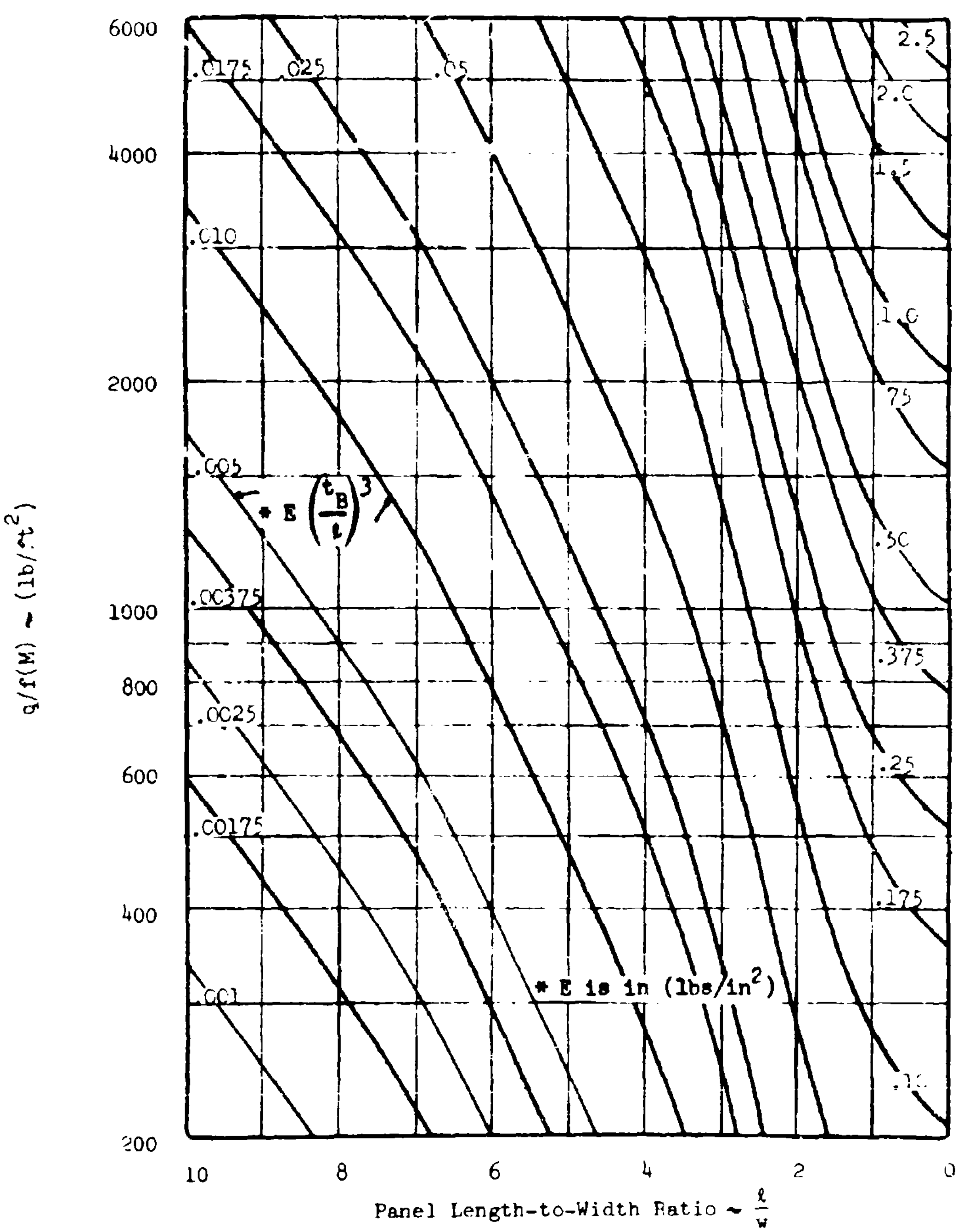


Figure 37- Aerodynamic Parameter $q/f(M)$ versus ℓ/w with Variation in Structural Parameter $E\left(\frac{t_B}{\ell}\right)^3$

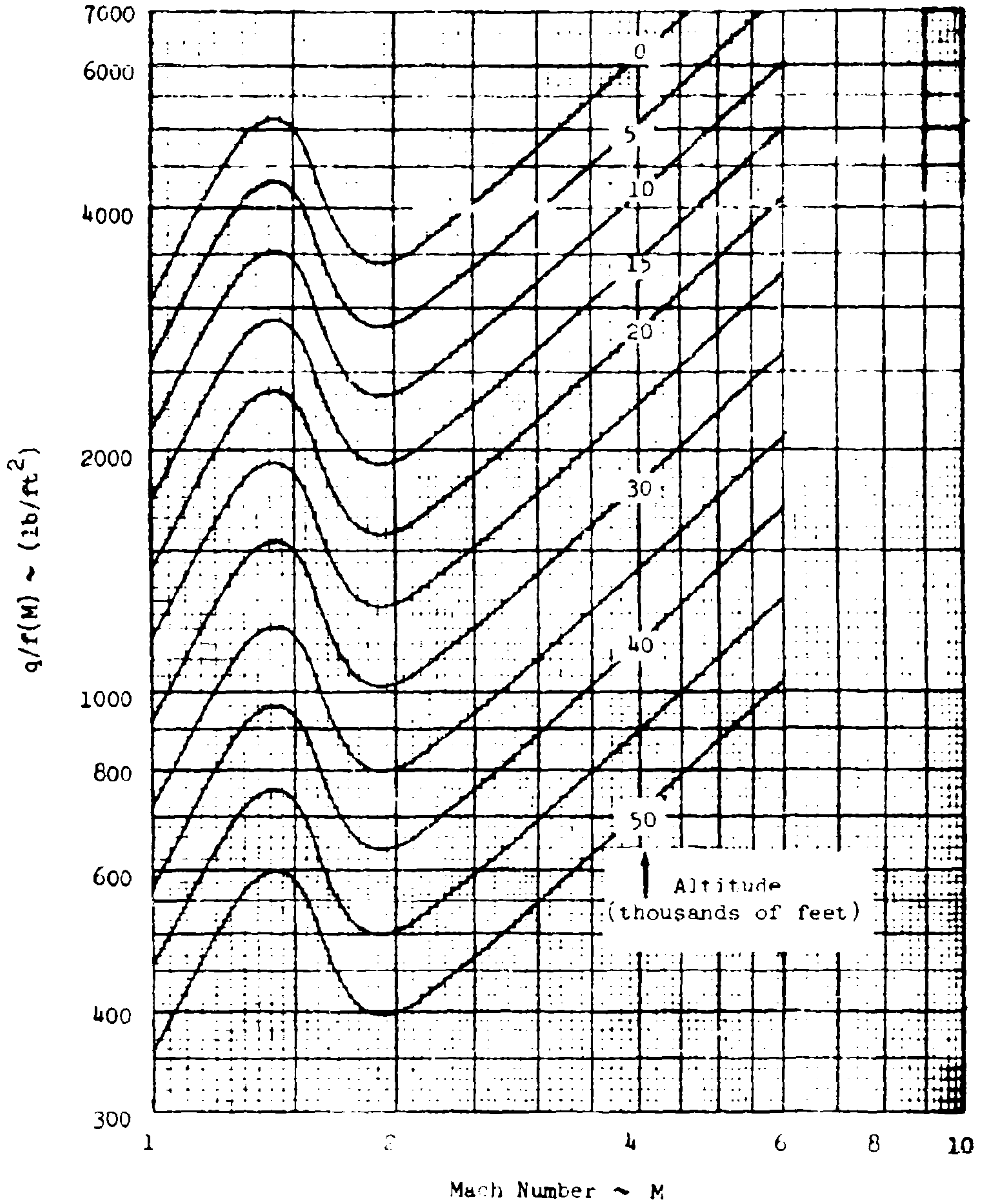


Figure 38- Plot of q/f(M) Versus Mach Number with Parametric Variation in Altitude

(3) Theoretical data are used exclusively where experimental data are lacking.

2. Supplemental Analyses

The design criteria that are presented in this report have been based, whenever possible, on experimental data. Several of the parameters however, have not been investigated in sufficient depth to define the thickness correction factors from experimental data alone. This Section describes the analytical effort that was used to obtain the required thickness correction factors. It is re-emphasized that the flutter boundary data obtained from these studies is used to determine the effect of parameter variation only in relation to the baseline flutter speed; the data are not used to obtain absolute flutter speeds.

Analysis of flutter behavior was obtained by using a modal approach that requires still air frequencies as input data. Therefore, there were two phases of analysis: the first determined the manner in which panel frequencies were expected to vary with the parameters of interest and the second phase was the flutter study that incorporated the still air frequencies.

a. Frequency Analysis

Modal frequency variation is considered to be the dominant cause of changes in flutter speed for the following parameters:

Length-to-width ratio
Curvature
Inplane stress
Temperature differential
Pressure differential
Enclosed cavity.

Of these, only curvature, pressure differential, and enclosed cavity were not well covered in the literature and hence required supplemental frequency analyses.

It was first necessary to define the modes that would participate in flutter. Supersonic point function aerodynamic theories indicate (and it is a fairly well accepted fact) that aerodynamic coupling can only exist between families of modes defined as follows: if m denotes the number of stream-wise half waves of a mode and n denotes the number of cross-stream half waves, then modes can only couple if they have a common n. Figure 40 shows certain groups of modes that may couple (node lines are dashed) for two airstream orientations.

The only modes that need be considered in most cases are those of the family m-1 and the modes 1-1, 2-1, 3-1, 4-1 were most commonly used. The determination of modal frequencies resulting from cylindrical curvature and cavity effect are discussed in Section III, parts 9 and 12, respectively. An analytical approach to account for differential pressure is given in Appendix B.

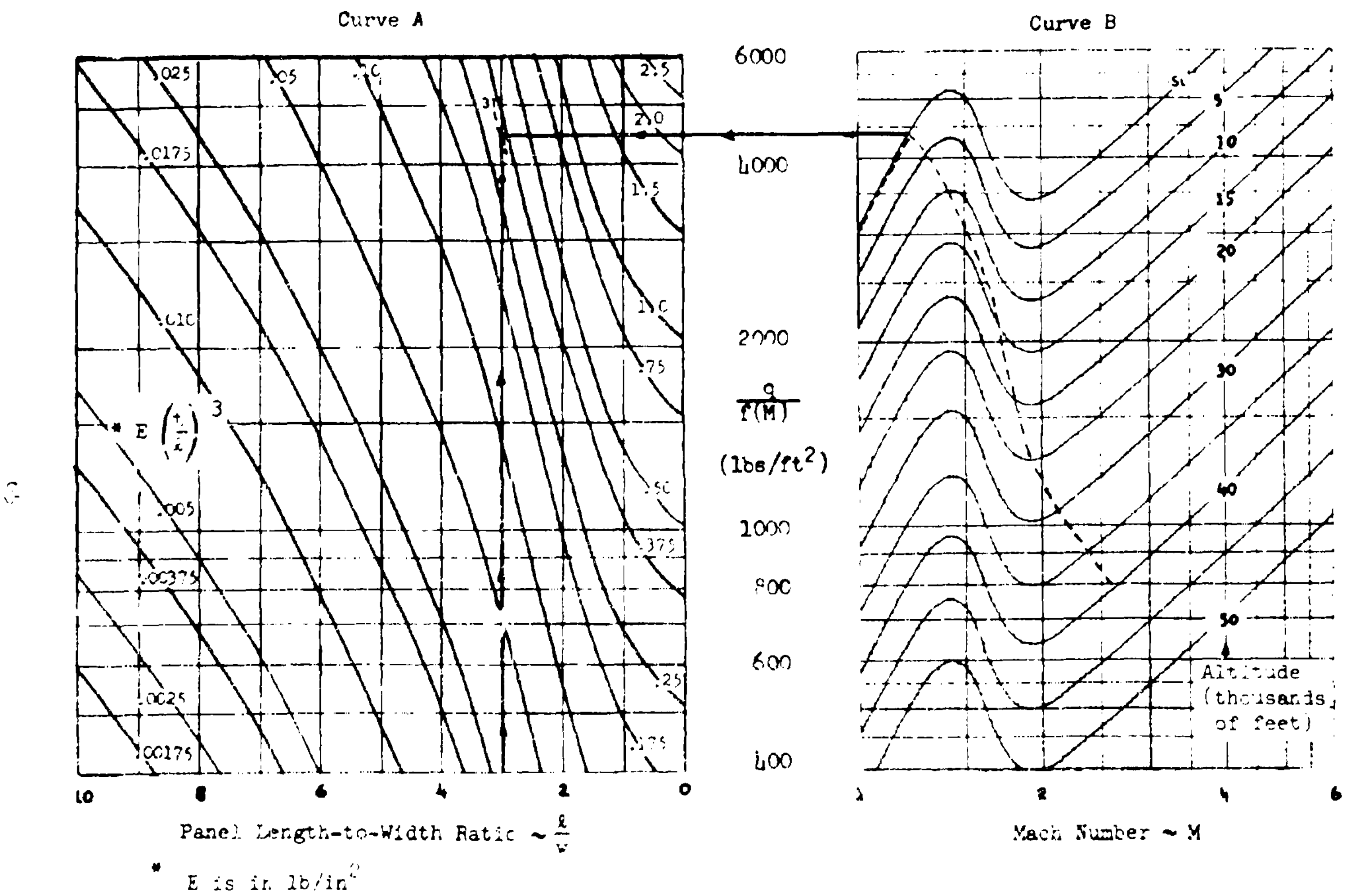


Figure 39 - Flat Panel Design Curves

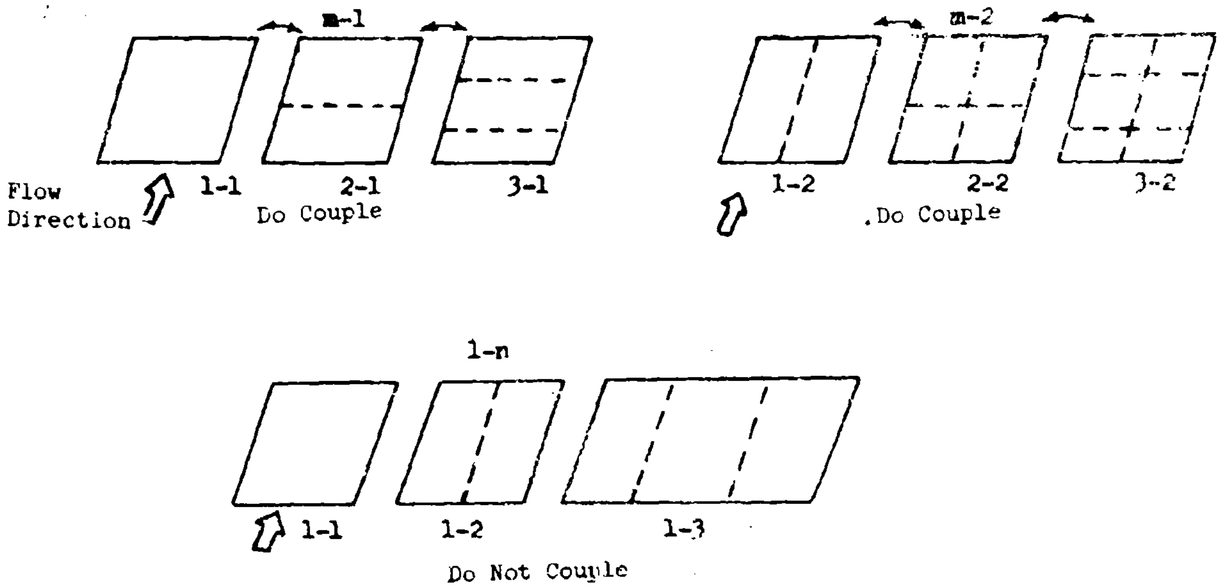


Figure 40 - Modal Families that May Couple Aerodynamically to Produce Flutter

b. Flutter Analysis

Four- and eight-mode aeroelastic studies, using static Ackeret aerodynamic theory, provided data that were required to establish flutter boundary trends. The assumed mode methods of analysis are well known and begin with a lateral displacement of the form:

$$w(x, y, t) = \sum_m \sum_n A_{mn} e^{i\omega t} F_m(x) G_n(y).$$

For simply-supported edges, the sine functions:

$$F_m(x) = \sin \frac{m\pi x}{\ell}$$

and

$$G_n(y) = \sin \frac{n\pi y}{w}$$

were used; for clamped edges, the convenient modes described by Warburton in Reference 39 were used. Lagranges' equations of motion, with appropriate nondimensioning, yields the flutter determinant

$$\left[C_{rm} - \gamma \delta_{rm} + \lambda B_{rm} = 0 \right]$$

For a clamped, rectangular panel of streamwise length ℓ, width, w, and mass density ρ_p, the elements of the determinant are

$$C_{rm} = k_{rm} \frac{\ell^3}{D_W}, \qquad \lambda = \frac{2\ell^3 q}{\beta D},$$

$$\delta_{rm} = \begin{matrix} 0, & r \neq m \\ 1, & r = m \end{matrix}, \qquad B_{rm} = \int_0^\ell \frac{\partial F_r}{\partial x} F_m dx,$$

$$\gamma = \frac{\rho_p t \ell^4 \omega^2}{D} .$$

The term k_{rm} is a stiffness, or elastic derivative that couples mode r-n with mode m-n. The expression γ is the eigenvalue and λ is a nondimensional dynamic pressure parameter. If r = m, the term C_{rr} is proportional to the natural frequency of mode r-n, i.e.

$$C_{rr} = \frac{\rho_p t \ell^4}{D} \omega_{rr}^2 .$$

The term B_{rm} is the aerodynamic derivative. Solutions were obtained by a method of trial and error in which the first coalescence of a pair of eigenvalues (i.e. $\gamma_1 = \gamma_2$) defined the flutter boundary, λ_{cr}. This detail is given to illustrate the manner in which the parameters were handled. If a parameter causes a change in modal frequency but not in mode shape, the effect on flutter speed can be determined by making the indicated changes in the diagonal elements C_{rr}. This method was adequate to study the effects of curvature and enclosed cavity. If on the other hand, a parameter causes a change in modal frequency and also mode shape, it is expedient to recalculate all elastic constants C_{rm}

both on and off the diagonal. This was done in studying Δp because of the non-uniform, static membrane stress. Critical values of λ were then found in the same way as previously described.

The flutter studies provided values of λ_{cr} as a function of a change in the parameter being investigated. The thickness correction factors t_i/t_B which are referenced to the baseline design were found as follows:

The modal flutter study is made for the flat panel configuration (i.e. modal frequencies not yet affected by parameters) and yields

$$\lambda_B = \frac{2\ell^3}{\beta D_B} q_B$$

The thickness of the panel is arbitrarily assigned the baseline value t_B. The parameter of interest is applied in the amount Δ to modify the flutter determinant and a new flutter solution gives

$$\lambda_\Delta = \frac{2\ell^3}{\beta D_\Delta} q_\Delta$$

We really want to know, however, the panel thickness t_Δ that is required for neutral stability at the original dynamic pressure q_B. Let

$$\lambda_\Delta = \frac{2\ell^3}{\beta D_\Delta} q_B$$

or from the flat panel boundary

$$\lambda_\Delta = \frac{2\ell^3}{\beta D_\Delta} \left(\lambda_B \frac{\beta D_B}{2\ell^3}\right)$$

For

$$D_B = \frac{E}{12(1-\nu^2)} t_B^3$$

and

$$D_{\Delta} = \frac{E}{12(1-\nu^2)} t_{\Delta}^3$$

and the parameters ℓ, β, E, and ν unchanged, the required thickness correction factor is

$$\frac{t_{\Delta}}{t_B} = (\frac{\lambda_B}{\lambda_{\Delta}})^{\frac{1}{3}}$$

REFERENCES

1. Jordan, P., "The Physical Nature of Panel Flutter," Aero Digest, pp. 34-38, February 1956.

2. Isaacs, R. P., "Transtability Flutter of Supersonic Aircraft Panels," Rand Report P-101, Rand Corporation, 1 July 1949.

3. Miles, J. W., "Dynamic Chordwise Stability at Supersonic Speeds," North American Aviation Report AL-1140, October 1950.

4. Shen, S. F., "Non-Stationary Aerodynamics of a Two-Dimensional Bump in a Uniform Stream and Its Effect on the Vibration Characteristics of an Elastic Panel," MIT Tech. Report. (Contract No. N5 ori-07833, NR 0640259 May 1952).

5. Shen, S. F., "Flutter of a Two-Dimensional Simply-Supported Uniform Panel in a Supersonic Stream," Massachusetts Institute of Technology, Aeroelasticity and Structures Laboratory, Report No. 25.10, 1952.

6. Sylvester, M. A. and Baker, J. E., "Some Experimental Studies of Panel Flutter at Mach Number 1.3," NACA TN 3914, 1957.

7. Goland, M. and Luke, Y. L., "An Exact Solution for Two-Dimensional Linear Flutter at Supersonic Speeds," Journal of Aeronautical Sciences, 21, 4, pp. 275-276, April 1954.

8. Hedgepeth, J. M., Budiansky, B. and Leonard, R. W., "Analysis of Flutter in Compressible Flow of a Panel on Many Supports," Journal of Aeronautical Sciences, 21, 7, pp. 4750486, July 1954.

9. Ashley, H. and Zartarian, G., "Piston Theory - A New Aerodynamic Tool for the Aeroelastician," Journal of Aeronautical Sciences, 23, 12, pp. 1109-1118, December 1956.

10. Hedgepeth, J. M., "Flutter of Rectangular Simply Supported Panels at High Supersonic Speeds," Journal of Aeronautical Sciences, 24, 8, pp. 563-573, August 1957.

11. Cunningham, H. J., "Analysis of the Flutter of Flat Rectangular Panels on the Basis of Exact Three-Dimensional Linearized Supersonic Potential Flow," IAS Paper 63-22, January 1963.

12. Sylvester, M. A., "Experimental Studies of Flutter of Buckled Rectangular Panels at Mach Numbers from 1.2 to 3.0 Including Effects of Pressure Differential and of Panel Width-Length Ratio," NASA TN D-833, May 1961.

13. Eisley, J. G., "The Flutter of a Two-Dimensional Buckled Plate with Clamped Edges in a Supersonic Flow,: California Institute of Technology, Ph.D. Thesis, July 1956. Published as AFOSR TN 56-296, 1956.

14. Lock, M. H. and Fung. Y. C., "Comparative Experimental and Theoretical Studies of the Flutter of Flat Panels in a Low Supersonic Flow," AFOSR TN 670, California Institute of Technology, May 1961.

15. Anderson, W. J., "Experiments on the Flutter of Flat and Slightly Curved Panels at Mach Number 2.81," CIT(GAL)SM 62-34, June 1962.

16. Dowell, E. H. and Voss, H. M., "Experimental and Theoretical Panel Flutter Studies in the Mach Number Range of 1.0 to 5.0," Report No. ASD-TDR-63-449, June 1963.

17. Hess, R. W., "Reliminary Results of an Experimental Study of the Effects of Compressive Stress on Panel Flutter" LWP-177, NASA Langley Research Center, January 1966. (Limited Distribution).

18. Bohon, H. L. and Dixon, S. C., "Some Recent Developments in Flutter of Flat Panels," AIAA 5th Annual Structures and Materials Conference, April 1964.

19. Bohon, H. L. and Anderson, M. S., "The Role of Boundary Conditions on Flutter of Orthotropic Panels," Proceedings of the AIAA Symposium on Structural Dynamics and Aeroelasticity, Boston, Mass., August 1965.

20. Bohon, H. L., "Experimental Flutter Results for Corrugation-Stiffened Panels at a Mach Number of 3.0," NASA TN D-2293, May 1964.

21. Dixon, S. C. and Shore, C. P., "Effects of Differential Pressure, Thermal Stress, and Buckling on Flutter of Flat Panels with Length-Width Ratio of 20," NASA TN D-2047, December 1963.

22. Fung, Y. C., "Summary of the Theories and Experiments on Panel Flutter," California Institute of Technology (GAL) Report 60E01 (AFOSR TN 60-224), May 1960.

23. Dugundji, J., "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers," MIT (ASRL) TR 134-1, August 1965.

24. Gaspers, P. A., Jr., and Redd, Bass, "A Theoretical Analysis of the Flutter of Orthotropic Panels Exposed to a High Supersonic Stream of Arbitrary Direction," Ames Research Center, NASA-TN-D-3551, August 1966.

25. Cunningham, H. J., "Flutter Analysis of Flat Rectangular Panels Based on Three-Dimensional Supersonic Unsteady Potential Flow," NASA TR-256, February 1967.

26. Kordes, E. E., Tuovila, W. J. and Guy, L. D., "Flutter Research on Skin Panels," NASA Technical Note D-451, September 1960.

27. Hodson, C. H. and Stocker, J. E., "Commercial Supersonic Transport Panel Flutter Studies," RTD-TDR-63-4036, May 1964.

28. Hess, R. W. and Osborn, J. A., "Some Effects of Differential Pressure and Damping Compound on the Flutter of Flat Panels Similar to Panels on the S-IVB Stage", Langley Working Paper LWP-166, November 1965 (Limited Distribution).

29. Kobett, D. R. and Zeydel, E. F. E., "Research on Panel Flutter," NASA TN D-2227, November 1963.

30. Olsen, James J., "Local Aerodynamic Parameters For Supersonic and Hypersonic Flutter Analysis," USAF (FDL) TR-65-183, October 1965.

31. Dowell, E. H., "The Flutter of Very Low Aspect Ratio Panels," AFOSR 64-1723 (ASRL 112-2), July 1964.

32. Kordes, E. E. and Noll, R. B., "Theoretical Flutter Analyses of Flat Rectangular Panels in Uniform Coplanar Flow with Arbitrary Direction," NASA TN D-1156, January 1962.

33. Lemley, C. E., "The Effect of Inplane Edge Restrain on the Vibration and Flutter of Slightly Curved Panels," D.Sc. Thesis, Department of Applied Mechanics, Washington University, St. Louis, January 1967.

34. Donnell, L. H. "Stability of Thin-Walled Tubes Under Torsion," National Advisory Committee for Aeronautics Report No. 479, 1933.

35. Batdorf, S.B., "Critical Combinations of Longitudinal and Transverse Direct Stress for an Infinitely Long Flat Plate", ARR L6A05, 3-3-45.

36. Rich, R. L., "Preliminary Skin Panel Flutter", Boeing Document D2-8148, 26 January 1962.

37. Golden, C. T., Hager, T. R., and Sherman, L. L., "Orthotropic Panel Flutter Analysis Correlation", Boeing Document D2-81301, 1964.

38. Warburton, G. B., "The Vibration of Rectangular Plates", Proc. of the Inst. of Mech. Engineers, Vol. 168, pp. 371-384.

APPENDIX A

SURVEY INFORMATION

The government-industry-university survey proved to be invaluable in establishing the state of the art in panel flutter. Answers were primarily sought to the two questions "What effort is currently being carried on to extend knowledge in the field?" and "What criteria are in current use for the design of flutter free panels?" The authors believe that these questions were answered satisfactorily, and are deeply indebted to the persons who have kindly provided information and discussions of their efforts in the field. The facilities and persons contacted are listed in Part (1) of this Appendix. In Part (2), a brief discussion is presented of those facilities that are involved in current efforts or are currently designing panels to preclude flutter.

Part (1) - Facilities and Persons Contacted

Facilities	Persons
NASA/Langley Research Center	L. Guy Dr. M. Anderson S. C. Dixon C. P. Shore H. J. Cunningham G. Rainey R. W. Hess
Princeton University	Professor Earl Dowell
Martin Company, Baltimore	Dr. Peter Jordan R. Goldman J. Tomassoni
MIT	Professor J. Dugundji
Boeing Co., Seattle	L. L. Sherman W. Weatherill H. Voss
North American Aviation, (Los Angeles)	Harold Sweet C. R. Hodson J. R. Stevenson
North American Aviation, (Columbus)	L. Kazmerzak J. Murphy G. Cook
Lockheed - California Co.	E. E. Postel P. C. Durup

Facilities	Persons
Aerospace Corporation (El Segundo)	Dr. M. H. Lock
Northrop Corporation, Hawthorne	T. Rooney S. Schwartz
Douglas (Missiles & Space Division)	P. F. Spas (Santa Monica) D. Roudebush A. W. Trudell C. M. Fuller J. J. McLaren
University of Michigan	Professor W. J. Anderson
Midwest Research Institute	D. R. Kobett
NASA/Ames Research Center	P. Gaspers L. Muhlstein
NASA/Flight Research Center, EAFB	Dr. E. E. Kordes J. M. Groen R. E. Klein
University of Texas	Professor R. O. Stearman
U. S. Air Force, Flight Dynamics Laboratory	M. Shirk D. Cooley

Part (2) - Current Effort

NASA/Langley Research Center

Theoretical efforts by L. Guy and his colleagues are quite extensive and include:

- Strength of panel instabilities
- Effects of edge support flexibility
- Effects of damping

Experimental investigations include:

- Flutter of orthotropic panels with elastic side edge report
- Effects of damping and edge rotational restraint for stressed isotropic panels

NASA/Langley Research Center

G. Rainey and R. W. Hess are completing a thorough experimental/theoretical study of some thirteen flat panels. Flutter boundaries were obtained for unstressed and buckled panels of length-width ratio between 1 and 4.5. An interim report was released as a Langley Working Paper (LWP-177) and the final report should be published in 1968.

Martin Company (Baltimore)

John Tomassoni has developed panel design criteria for in-house use. The criteria account for Mach No., inplane stress, curvature, buckling, and are based on such documents as ARTC-32, LWP-177, and NASA TN 3781. The criteria apply to both flat and curved buckled panels and were formulated for the use of designers.

Boeing Company (Seattle)

L. L. Sherman has based panel design effort on NASA TN D-833, TN D-1386, TN D-1156, together with personal experience gained from early tests of Dyna Soar panels. He has used the panel design concept that is employed in this report, that is, he first formulates a flat, unstressed panel design and then specifies thickness modifications to account for the parameters that affect flutter boundaries. He believes that reliable hardware design requires knowledge of the still-air panel dynamics.

NASA-Flight Research Center

J. M. Groen has recently conducted flight flutter tests of flat panels attached to a fixture that is suspended under the F-104 airplane. The purpose was to obtain data for panels that simulate the fabrication techniques of most aerospace applications. The result shows reasonable agreement with TN D-451 envelopes, although the panels at $\ell/w = 1$ were outside the envelope. A report should be released in 1968.

NASA/Ames Research Center

P. Gaspers and L. Muhlstein are conducting in-house effort in panel flutter research. Results of recently completed investigation of boundary layer (to be published) indicate a pronounced stabilizing effect with optimum effectivity occurring at $M = 1.2$. Flow angularity data have been obtained but will be published later than the boundary layer investigation.

Lockheed-California Company

E. E. Postel proposes use of a two-degree-of-freedom design approach based on still-air natural mode frequencies (reported in Lockheed Report No. LR 17961). P. C. Durup has recently formulated panel flutter criteria based on TN D-1949, TN D-451, TN D-1386, and some unpublished data obtained from NASA/Ames.

APPENDIX B

DIFFERENTIAL PRESSURE ANALYSIS

The approximate analysis that is presented here was made for two purposes: the first was to determine the form of a nondimensional parameter involving Δp that would describe its effect on a flat, uniform panel, and the second purpose was to obtain an analytical method that allows a prediction of panel dynamics so that the flutter speed can then be obtained. The procedure involved two steps and was performed as follows:

(a) Determine the distribution of static inplane stress that is a function of the applied differential pressure. Assume that the edges are simply supported in rotation and that the uniform pressure load Δp deforms the panel into a static shape

$$w_s = d_o \sin \frac{\pi x}{\ell} \sin \frac{\pi y}{w} \tag{1}$$

in which the peak static deflection d_o is a nonlinear function of Δp. The opposite edges are assumed to remain a constant distance apart so that the static mode shape can only be accommodated by inplane stretch. The sketch in Figure 41 shows the coordinate system used for the analysis.

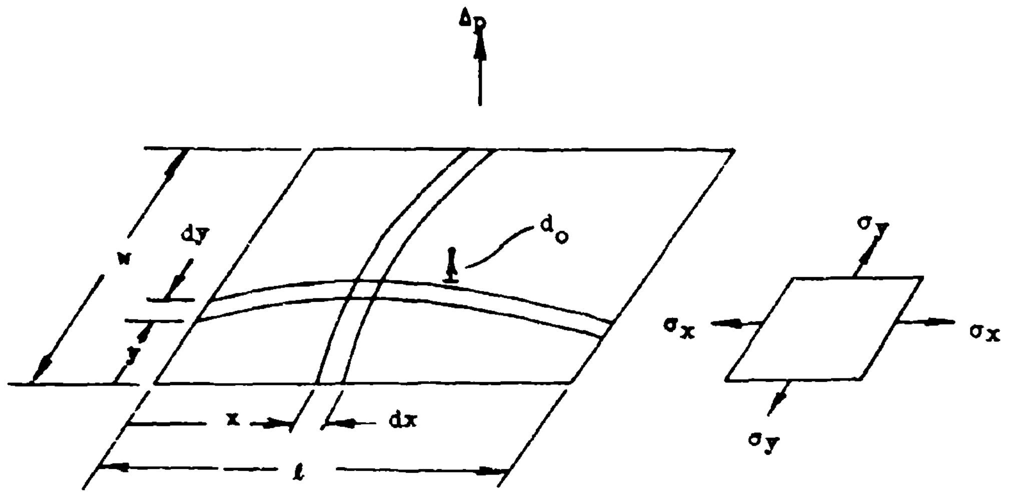


Figure 41 - Coordinate System Used to Analyze Static Effect of Δp

The axial strains are ϵ_x and ϵ_y; the x-direction strain ϵ_x has the form

$$\epsilon_x = \frac{1}{2\ell} \int_0^{\ell} \left(\frac{\partial w_s}{\partial x}\right)^2 dx \tag{2}$$

which assumes that the strain is constant between $x = 0$ and $x = \ell$ but varies with the coordinate y. Likewise

$$\epsilon_y = \frac{1}{2w} \int_0^{w} \left(\frac{\partial w_s}{\partial y}\right)^2 dy \tag{3}$$

with similar assumptions. Since $\sigma_z = 0$, the generalized Hooke's law yields

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu\epsilon_y) \tag{4}$$

and

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu\epsilon_x) \tag{5}$$

Substitute Equation (1) into Equations (2) and (3), evaluate the integrals; then substitute for ϵ_x and ϵ_y into (4) and (5) to get

$$N_x = \sigma_x t = \frac{Et}{1-\nu^2} \left[\frac{1}{2} \left(\frac{d_o}{t}\right) \pi \left(\frac{t}{\ell}\right)\right]^2 \times \left[\sin^2 \left(\frac{\pi y}{w}\right) + \left(\frac{\ell}{w}\right)^2 \sin^2 \frac{\pi x}{\ell}\right] \tag{6}$$

$$N_y = \sigma_y t = \frac{Et}{1-\nu^2} \left[\frac{1}{2} \left(\frac{d_o}{t}\right) \pi \left(\frac{t}{\ell}\right)\right]^2 \times \left[\left(\frac{\ell}{w}\right)^2 \sin^2 \frac{\pi x}{\ell} + \sin^2 \frac{\pi y}{w}\right] \tag{7}$$

The information yet required is the manner in which the differential pressure and the crown height are related, i.e.,

$$\Delta p = f(d_o)$$

and the frequency relationship

$$\omega_{mn} = f(\Delta p)$$

The first of these is obtained by an external work-strain energy relationship

$$\delta W_{ex} = \delta U_{int}$$

in which W_{ext} is the work done by the pressure Δp in deforming the panel, and U_{int} is the sum of the bending U_B and stretching U_M energies of the deformed panel. The variational equation leads to the static form of Lagrange's equation

$$\frac{\partial W_{ex}}{\partial d_o} = \frac{\partial U_B}{\partial d_o} + \frac{\partial U_M}{\partial d_o} .$$

The component parts of this relationship are obtained from

$$\delta W_{ex} = \int_0^{\ell}\int_0^{w} \Delta p \; \delta w_s \; dx \; dy$$

in which

$$\delta w_s = \delta \; d_o \sin \frac{\pi x}{\ell} \sin \frac{\pi y}{w}$$

so that

$$\frac{\partial W_{ex}}{\partial d_o} = 0.406 \; \Delta p w \ell . \tag{8}$$

$$U_B = \int_0^{\ell}\int_0^{w} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 dx \; dy$$

leads to (for $\nu = 0.3$)

$$\frac{\partial U_B}{\partial d_o} = 2.23 \; \frac{Et^3 w d_o}{\ell^3} \left[1 + \left(\frac{\ell}{w}\right)^2\right]^2 \tag{9}$$

$$U_m = \frac{Et}{2(1-\nu^2)} \int_0^{\ell}\int_0^{w} (\epsilon_x^2 + \epsilon_y^2 + 2 \nu \epsilon_x \epsilon_y) dx \; dy$$

which becomes

$$\frac{\partial U_m}{\partial d_o} = 5.08 \; \frac{Etw d_o^{\,3}}{\ell^3} \left[1 + .4 \left(\frac{\ell}{w}\right)^2 + \left(\frac{\ell}{w}\right)^4\right] \tag{10}$$

Therefore, the required relation is

$$0.183 \left[\frac{\Delta p}{E(\frac{t}{\ell})^4}\right] = \frac{d_o}{t}\left[1 + (\frac{\ell}{w})^2\right]^2$$

$$+ 2.28 \, (\frac{d_o}{t})^3\left[1 + .4(\frac{\ell}{w})^2 + (\frac{\ell}{w})^4\right] \qquad (11)$$

This relationship has been plotted in Figure 32 as the nondimensional parameter $\Delta p/E\left[\frac{t}{\ell}\right]^4$ versus ℓ/w with variation in the crown height parameter d_o/t. Note that d_o/t is a measure of the amount of panel deformation d_o in relation to the panel thickness t.

It is convenient to perform the dynamic (vibration and flutter) portion of the analysis in terms of d_o/t, as noted by the form of Equations (6) and (7). Given the form of N_x and N_y, four- and eight-mode vibration and flutter analyses were made. The form of the inplane loads, in Figure 42 below,

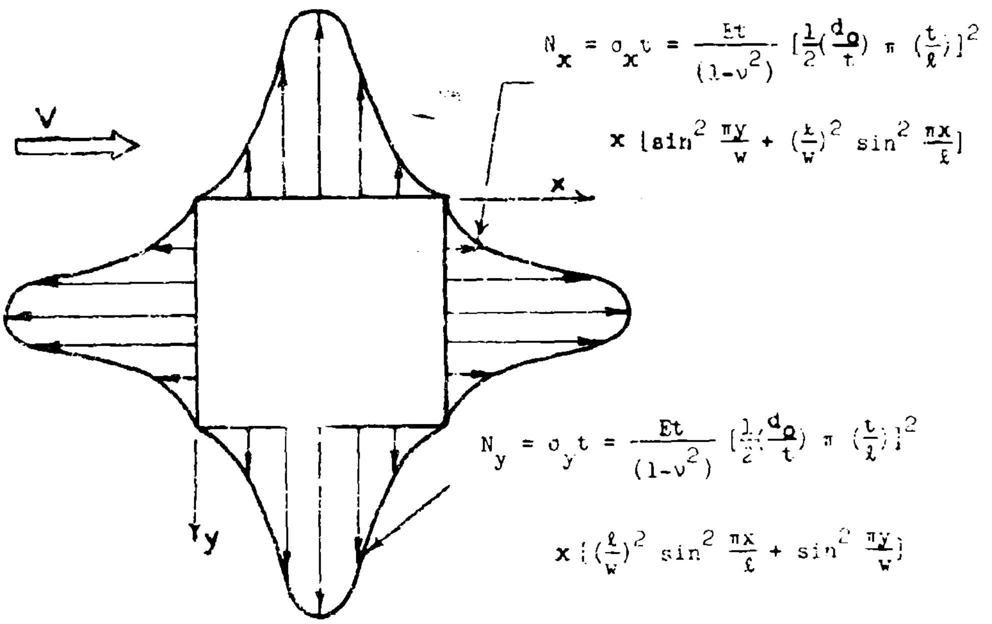


Figure 42 - Static Inplane Loading That Results From Δp

caused the evaluation of energies to be quite tedious.

The flutter characteristics of the simply supported panel shown in Figure 46 were obtained from Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_n}\right) + \frac{\partial U}{\partial q_n} = Q_n.$$

where the kinetic energy T, potential energy U, and generalized forces Q_n, are functions of the generalized coordinates q_n.

The kinetic energy of the panel is given by

$$T = \frac{1}{2}\int_0^{\ell}\int_0^{w} \rho_m t\left(\frac{\partial w}{\partial t}\right)^2 dx\, dy.$$

The potential energy of the panel results from bending of the panel U_B and the energy contribution of the inplane loading U_M. The energy expressions corresponding the potential energy terms are given by

$$U_B = \frac{D}{2}\int_0^{\ell}\int_0^{w}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 dx\, dy$$

and

$$U_M = \frac{1}{2}\int_0^{\ell}\int_0^{w}\left(N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2\right) dx\, dy.$$

Assuming the aerodynamic loading on the panel can be represented by the Ackeret value

$$p(x,y,t) = -\frac{2q}{\beta}\frac{\partial w}{\partial x}$$

where $\beta = \sqrt{M^2-1}$, q is the free stream dynamic pressure and $\frac{\partial w}{\partial x}$ is the slope of the panel in the stream direction. The virtual work δW_A associated with the above aerodynamic loading for a virtual displacement δw is given by .

$$\delta W_A = \frac{2q}{\beta}\int_0^{\ell}\int_0^{w}\left(\frac{\partial w}{\partial x}\right)(\delta w)\, dx\, dy$$

The generalized force term associated with the aerodynamic loading can then be found from

$$\delta W_A = Q_A \delta q$$

where Q_A is the generalized force and δq is the virtual displacement of the generalized coordinate q.

The assumption is now made that panel deflections associated with the kinetic energy, potential energy, and generalized force can be described; with sufficient accuracy, by

$$w = \sum_m \sum_n A_{mn} e^{i\omega t} \sin\left(\frac{m\pi x}{\ell}\right) \sin\left(\frac{n\pi y}{w}\right)$$

where the time varying function $e^{i\omega t}$ denotes simple harmonic motion and the generalized coordinates are A_{mn} (m and n represent the number of half sine waves in the stream and cross-stream directions, respectively).

Evaluating the integrals required for Lagrange's equations, and solving the flutter determinant for q = 0 (still air) leads to the frequency behavior shown in Figures 43, 44, 45, 46, and 47 for the first four stream wise modes. These plots are for length-width ratios ℓ/w = 1/2, 1, 2, 3, and 4 and the frequencies are plotted against the crown height parameter d_o/t. The frequency behavior is related back to the pressure parameter $\frac{\Delta p}{E(\frac{t}{\ell})^4}$ with the aid of Figure 32. The flutter solutions [obtained in the manner described in Section IV (Supplemental Analyses)], yields the flutter frequencies that are plotted as the dashed lines on Figures 43 thru 47, with the theoretical thickness factor being presented in Figure 33.

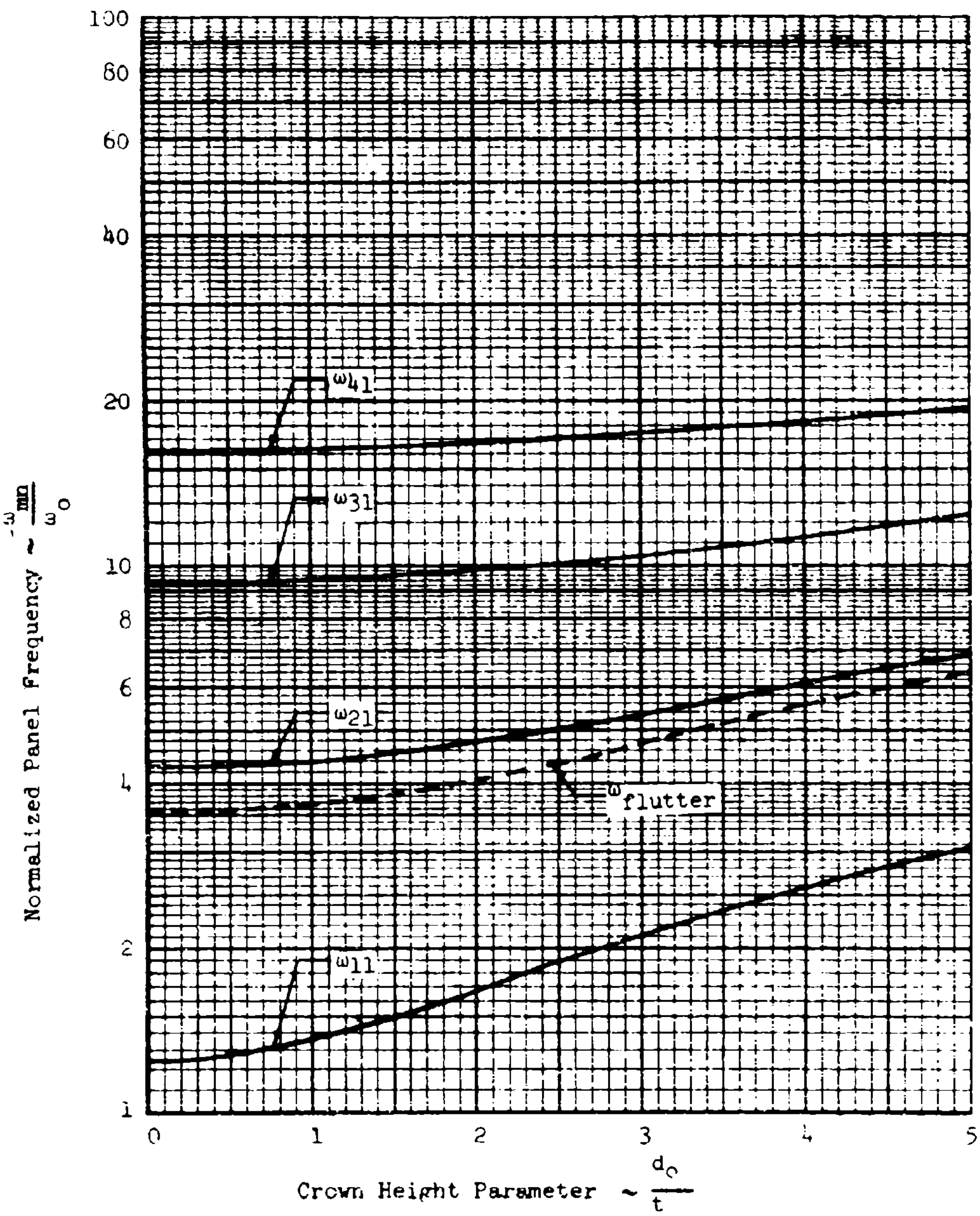


Figure 49 Normalized Panel Frequencies Versus Crown Height Parameter for Length-to-Width Ratio of One-Half

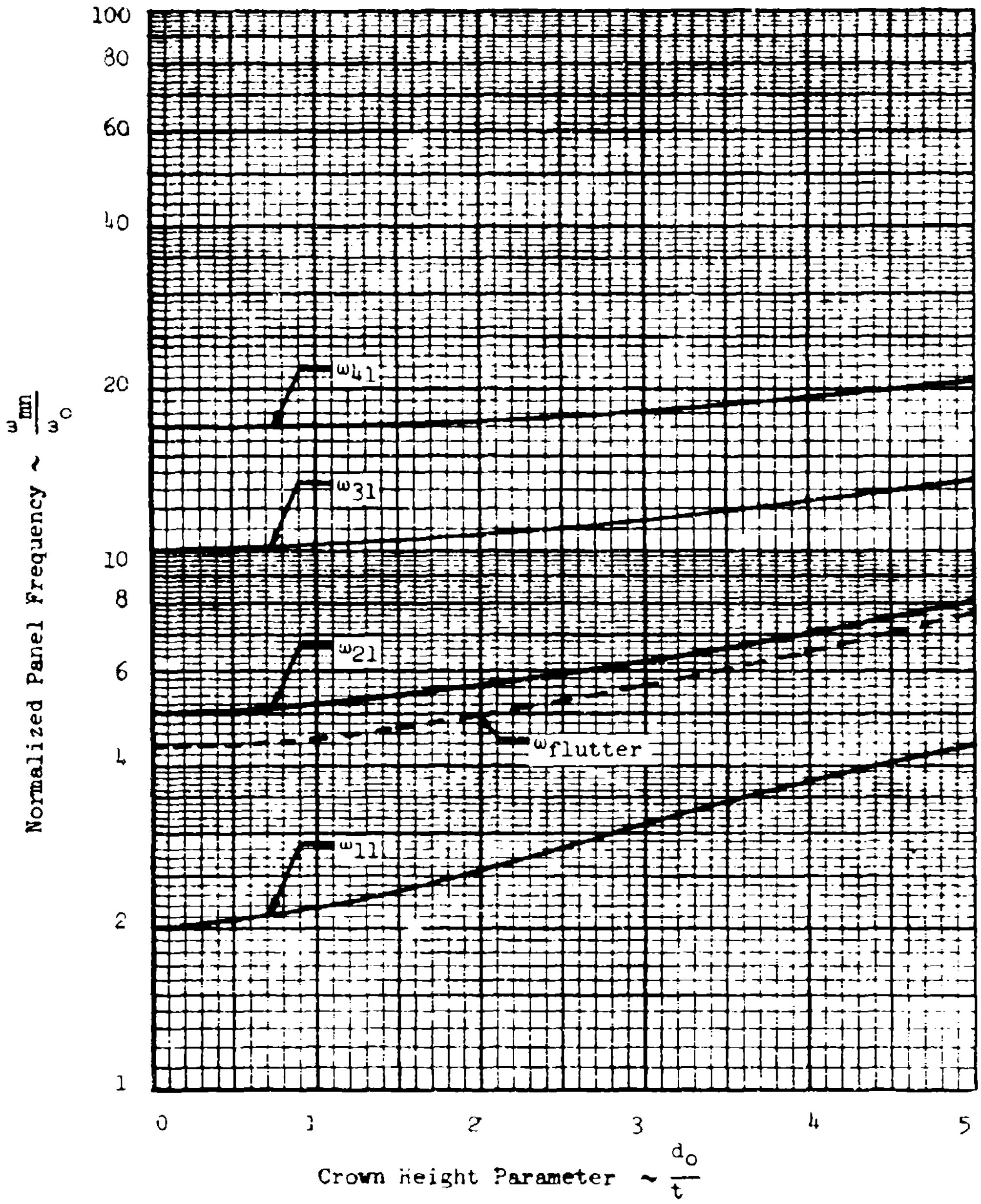


Figure 41.- Normalized Panel Frequencies Versus Crown Height Parameter for Length-to-Width Ratio of One

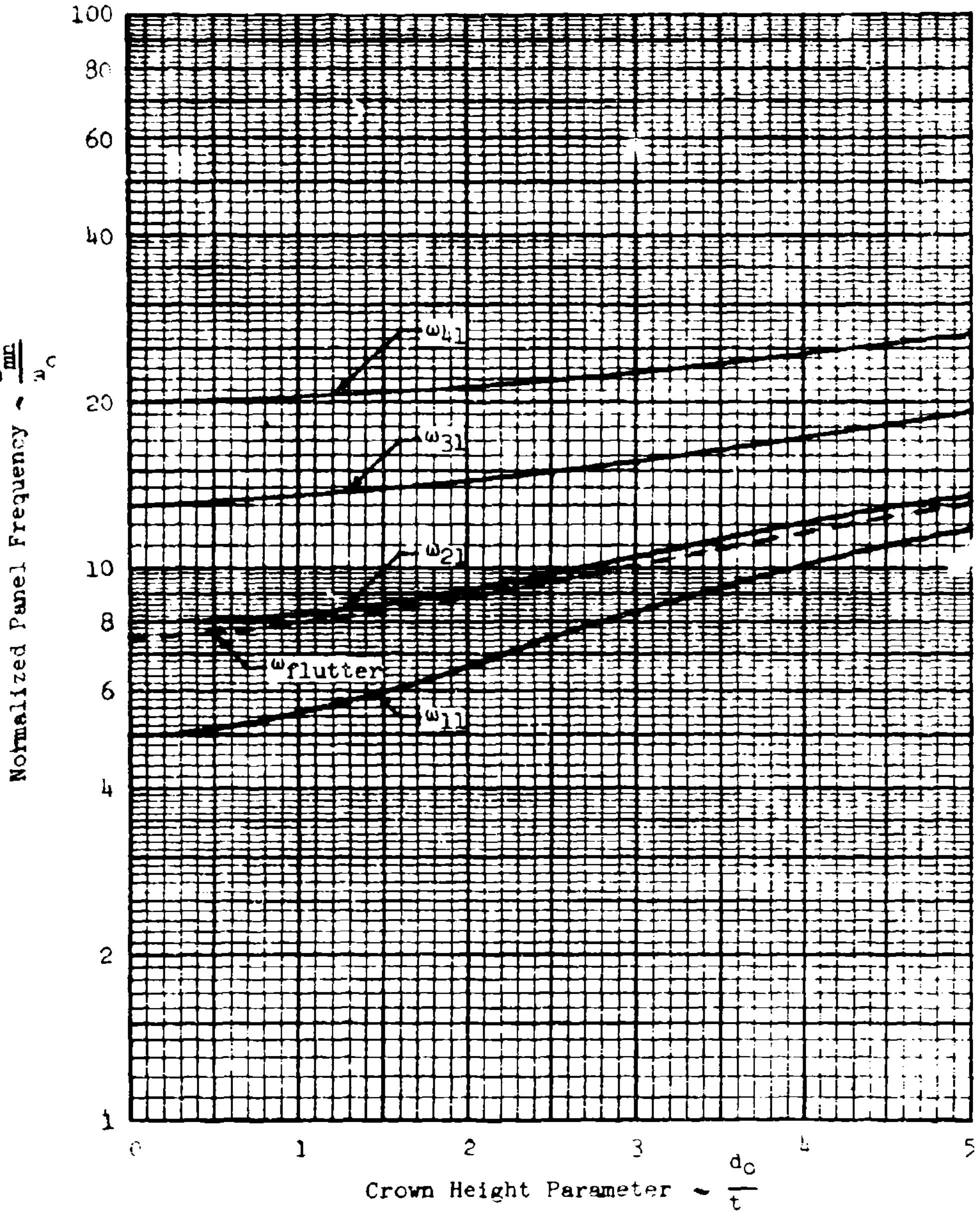


Figure 45- Normalized Panel Frequencies Versus Crown Height Parameter for Length-to-Width Ratio of Two

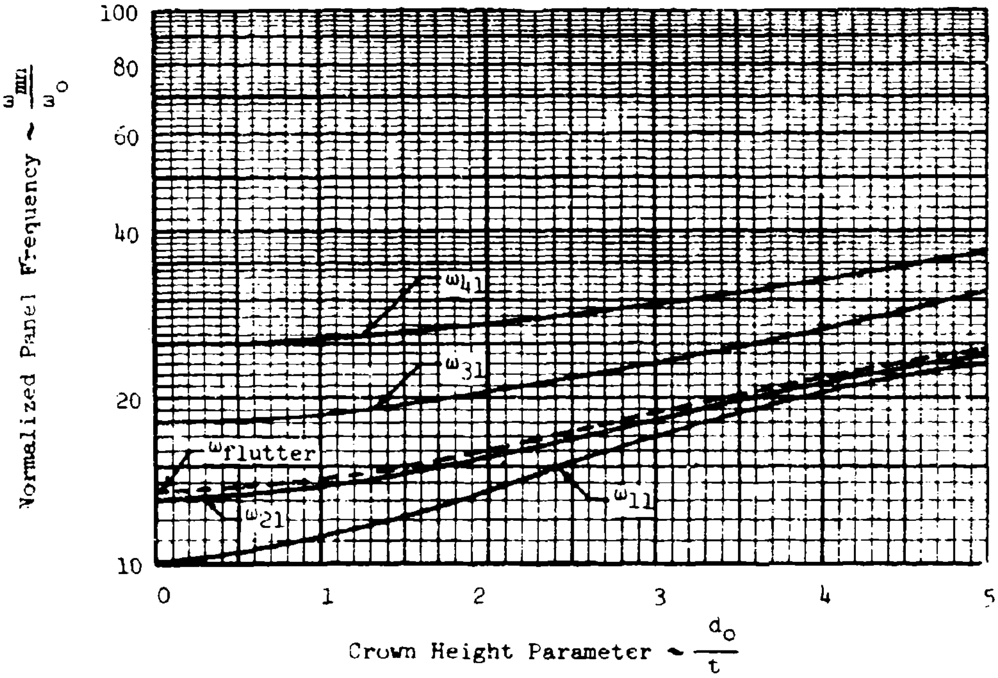


Figure 46- Normalized Panel Frequencies Versus Crown Height Parameter for Length-to-Width Ratio of Three

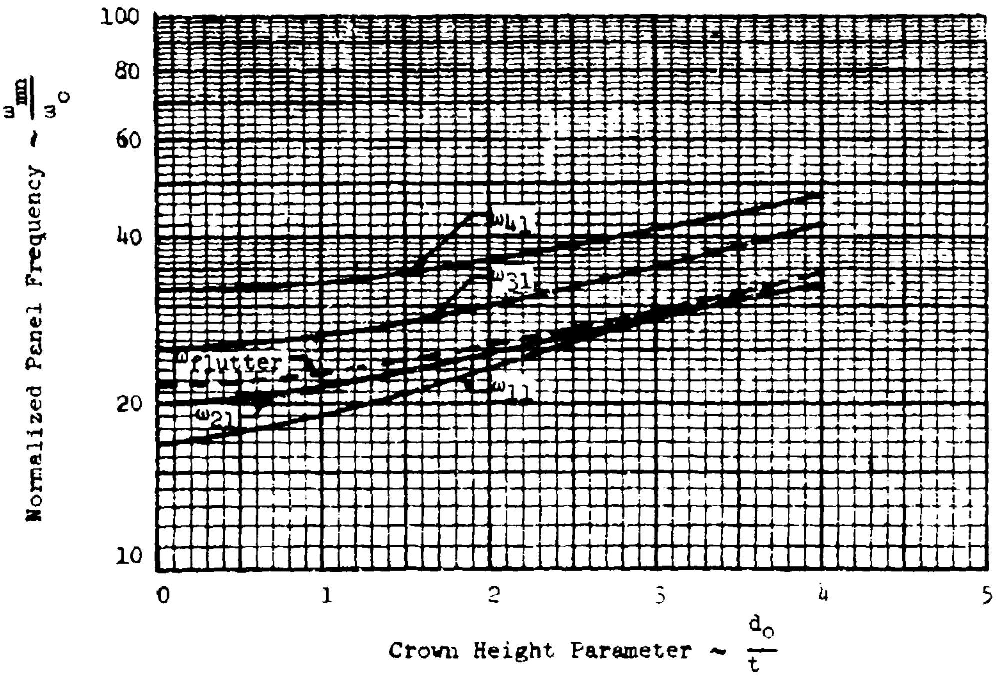


Figure 47 - Normalized Panel Frequencies Versus Crown Height Parameter for Length-to-Width Ratio of Four

APPENDIX C

PANEL FLUTTER BIBLIOGRAPHY

The literature on the subject of panel flutter, and in related areas (viz., plate and shell dynamics, structures, aerodynamics, boundary layer phenomena) is very extensive and the writers cannot be sure that all papers of significance have been included. It is believed, however, that the literature reviewed, as indicated in the bibliography, fairly represents the existing state-of-the-art.

1. Anderson, W. J., "Experiments on the Flutter of Flat and Slightly Curved Panels at Mach Number 2.81," CIT(GAL)SM 62-34, June 1962.

2. Anderson, W. J. and Fung, Y. C., "The Effect of an Idealized Boundary Layer on Flutter of Cylindrical Shells in Supersonic Flow," CIT(GAL)SM 62-49, December 1962.

3. Anderson, W. J., "Oscillatory Pressures in an Idealized Boundary Layer with and Application to the Panel Flutter of Cylindrical Shells," Proceedings of the AIAA Symposium on Structural Dynamics and Aeroelasticity, Boston, Mass., August 1965.

4. Asher, G. W. and Brown, A. W., "Experimental Studies of the Unsteady Aerodynamics of Panels At or Near Flutter with a Finite Boundary Layer Mach Number 1 to 10," The Boeing Company, RTD-TDR-63-4268, December 1964.

5. Ashley, H. and Zartarian, G., "Piston Theory - A New Aerodynamic Tool for the Aeroelastician," Journal of Aeronautical Sciences, 23, 12, pp. 1109-1118, December 1956.

6. Becker, Herbert, "Thermoelastic Stress Concentrations," AIAA Paper 66-120, January 24 - 26, 1966.

7. Beis, D. A., "A Wind Tunnel Investigation of Panel Response to Boundary Layer Pressure Fluctuations at Mach 1.4 and 3.5," NASA CR-501, May 1966.

8. Bohon, H. L. and Anderson, M. S., "The Role of Boundary Conditions on Flutter of Orthotropic Panels," Proceedings of the AIAA Symposium on Structural Dynamics and Aeroelasticity, Boston, Mass., August 1965.

9. Bohon, H. L., "Experimental Flutter Results for Corrugation-Stiffened Panels at a Mach Number of 3.0," NASA TN D-2293, May 1964.

10. Bohon, H. L. and Dixon, S. C., "Some Recent Developments in Flutter of Flat Panels," AIAA 5th Annual Structures and Materials Conference, April 1964.

11. Bohon, H. L., "Flutter of Flat Rectangular Orthotropic Panels with Biaxial Loading and Arbitrary Flow Direction," NASA TN D-1949, September 1963.

12. Bohon, H. L., "Panel Flutter Tests on Full Scale X-15 Lower Vertical Stabilizer at Mach Number of 3.0," NASA TN-1385, October 1962.

13. Bolotin, V. V., Gavrilow, Iu. V., Makarov, B. P. and Shreikl, Iu. Iu., "The Nonlinear Problem of the Stability of Flat Panels in High Supersonic Velocity Flow,: Akad, Nauk USSR, Otd, Tekn, Nauk, Izvestia Mekh, i Mashinostr (in Russian), pp. 59-64, May-June 1959. (Ref. from Journal of Aero/Space Engineering, October 1959.) (Rectangular Plates, M > 1. Solution verified on electronic computer.)

14. Brown, R. A., "Frequency Effects in Panel Flutter of Cylindrical Shells," University of California (IER) Rpt. AS-64-6, March 1964.

15. Calligeros, J. M. and Dugundji, J., "Effects of Orthotropicity Orientation on Supersonic Panel Flutter," AIAA Journal Vol. 1, No. 9, September 1963.

16. Calligeros, J. M. and Dugundji, J., "Supersonic Flutter of Rectangular Orthotropic Panels with Arbitrary Orientation of Orthotropicity," AFOSR 5328, June 1963.

17. Courant, R., "Variational Methods for the Solution of Problems of Equilibrium and Vibrations," American Mathematical Society Bulletin, 49, pp. 1-23 January 1943. See also, R. Courant and D. Hilbert, "Methods of Mathematical Physics," p. 208, Interscience Publishers, 1953.

18. Crisp, J. D. C., "The Equation of Energy Balance for Fluttering Systems with Some Applications in the Supersonic Regime," Journal of Aerospace Sciences, 26, 11, pp. 703-716, November 1959.

19. Cummings, Benjamin E., "Supersonic Flutter of a Plate Containing Many Bays in a Spanwise Direction," California Institute of Technology, Aero E. Thesis, June 1957.

20. Cummings, B. E., "Large Amplitude Vibration and Response of Curved Panels," AIAA Summer Meeting, June 17-20, 1963.

21. Cunningham, H. J., "Analysis of the Flutter of Flat Rectangular Panels on the Basis of Exact Three-Dimensional Linearized Supersonic Potential Flow," IAS Paper 63-22, January 1963.

22. Cunningham, H. J., "Flutter Analysis of Flat Rectangular Panels Based on Three-Dimensional Supersonic Unsteady Potential Flow," NASA TR R-256, February 1967.

23. Dixon, S. C., "Experimental Investigation at Mach Number 3.0 of Effects of Thermal Stress and Buckling on Flutter Characteristics of Flat Single-Bay Panels of Length-Width Ratio 0.96," NASA TN D-1485, November 1962.

24. Dixon, S. C., "Application of Transtability Concept to Flutter of Finite Panels and Experimental Results," NASA TN D-1948, September 1963.

25. Dixon, S. C., Griffith, G. E. and Bohon, H. L., "Experimental Investigation at Macn Number 3.0 of the Effects of Thermal Stress and Buckling on the Flutter of Four-Bay Aluminum Alloy Panels with Length-Width Ratios of 10," NASA TN-921, October 1961.

26. Dixon, S. C. and Shore, C. P., "Effects of Differential Pressure, Thermal Stress, and Buckling on Flutter of Flat Panels with Length-Width Ratio of 20," NASA TN D-2047, December 1963.

27. Dixon, S. C., "Comparison of Panel Flutter Results from Approximate Aerodynamic Theory with Results from Exact Inviscid Theory and Experiment," NASA TN D-3649, October 1966.

28. Dowell, E. H. and Voss, H. M., "Experimental and Theoretical Panel Flutter Studies in the Mach Number Range of 1.0 to 5.0," Report No. ASD-TDR-63-449, June 1963.

29. Dowell, E. H. and Voss, H. M., "Experimental and Theoretical Panel Flutter Studies in the Mach Number Range of 1.0 to 5.0," Report No. ASD-TDR-63-449, Supplement 1, 1965.

30. Dowell, E. H., "The Flutter of Infinitely Long Plates and Shells," Proceedings of the AIAA Symposium on Structural Dynamics and Aeroelasticity, Boston, Mass., August 1965.

31. Dowell, E. H., "The Flutter of Multi-bay Panels at High Supersonic Speeds," MIT (ASRL) TR 112-1, August 1963.

32. Dowell, E. H., "The Flutter of Very Low Aspect Ratio Panels," AFOSR 64-1723 (ASRL 112-2), July 1964.

33. Dowell, E. H., "Nonlinear Oscillations of a Fluttering Plate," Paper No. 66-79, Presented at 3rd Aerospace Sciences Meeting, New York, January 1966.

34. Dowell, E. H. and Voss, H. M., "The Effect of a Cavity on Panel Vibrations," AIAA Journal, Vol. 1, No. 2, 1963.

35. Dugundji, J., Dowell, E. H. and Perkin, B., "Subsonic Flutter of Panels on Continuous Elastic Foundations, Experiment and Theory," MIT (ASRL) TR 74-4, April 1962.

36. Dugundji, J., "Effects of Orthotropicity Orientation on Panel Flutter", AIAA Journal, September 1963.

37. Dugundji, J., "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers," MIT (ASRL) TR 134-1, August 1965.

38. Durvasula, Somayajulu, "Flutter of Simply-Supported Parallelogrammic Flat Panels in Supersonic Flow," AIAA Paper No. 66-474.

39. Eisley, J. G. and Luessen, G., "The Flutter of Thin Plates Under Combined Shear and Normal Edge Forces Including the Effects of Varying Sweepback," IAS Paper 62-90, June 1962.

40. Eisley, J. G., "The Flutter of a Two-Dimensional Buckled Plate with Clamped Edges in a Supersonic Flow,: California Institute of Technology, Ph.D. Thesis, July 1956. Published as AFOSR TN 56-296, 1956.

41. Eisley, J. G., "The Flutter of Simply Supported Rectangular Plates in Supersonic Flow," AFOSR TN 55-236, California Institute of Technology, July 1955.

42. Erickson, L. L., "Supersonic Flutter of Flat Rectangular Orthotropic Panels Elastically Restrained Against Edge Rotation," Langley Research Center, NASA TD-D-3500, August 1966.

43. Faure, Gerard, "Panel Flutter in the Transonic Range," X65-10293 Translation of "Flottements de panneaux en transsonique," LaRecherche Aeronauticque, No. 88, (May/June), pp. 49-58, NASA-TT-F-9160, November 1964.

44. Flugge, W., "Statik und Dynamik der Schalen,' Julius Springer, Berlin, 1934.

45. Fralich, R. W., "Postbuckling Effects on the Flutter of Simply-supported Rectangular Panels at Supersonic Speeds," NASA TN D-1615, March 1963.

46. Fung, Y. C., "On Corrugation-Stiffened Panels," CIT(GAL) SM 62-33, June 1962.

47. Fung, Y. C., "Some Recent Contributions to Panel Flutter Research," AIAA Journal, Vol. 1, No. 4, April 1963.

48. Fung, Y. C., "The Static Stability of a Two-Dimensional Curved Panel in a Supersonic Flow with an Application to Panel Flutter," Journal of Aeronautical Sciences, 21, 8, pp. 556-565, August 1954.

49. Fung, Y. C., "The Flutter of Simply Supported Rectangular Plates in a Supersonic Flow," California Institute of Technology Rpt. (GALCIT) 86-1 (AFOSR-TN-55-236), July 1955.

50. Fung, Y. C., "The Flutter of a Buckled Plate in a Supersonic Flow," AFOSR-TN-55-237, California Institue of Technology, July 1955.

51. Fung, Y. C., "On Panel Flutter," IAS p/p 749 (AFOSR TR 57-49), June 1957.

52. Fung, Y. C., "Flutter of Curved Plates with Edge Compression in a Supersonic Flow," Proceedings of the Midwest Conference on Fluid and Solid Mechanics, Ann Arbor, Mich., April 1-2, 1957. Published as AFOSR TN 57-187.

53. Fung, Y. C., "On Two-Dimensional Panel Flutter," Journal of Aeronautical Sciences, 25, 3, pp. 145-160, 1958.

54. Fung, Y. C., "Summary of the Theories and Experiments on Panel Flutter," California Institute of Technology (GAL) Report 60E01 (AFOSR TN 60-224), May 1960.

55. Garrick, J. E. and Rubinow, S. I., "Theoretical Study of Air Forces on an Oscillating or Steady Thin Wing in a Supersonic Main Stream," NACA Rpt. No. 872, 1946.

56. Garrick, J. E. and Rubinow, S. I., "Flutter and Oscillating Air Force Calculations for an Airfoil in a Two-Dimensional Supersonic Flow," NACA Rpt. No. 846, 1947.

57. Gaspers, P. A., Jr., and Redd, Bass, "A Theoretical Analysis of the Flutter of Orthotropic Panels Exposed to a High Supersonic Stream of Arbitrary Direction," Ames Research Center, NASA-TN-D-3551, August 1966.

58. Gerad, A., "Panel Flutter in the Transonic Range," (Translated from French), NASA TT F-9160.

59. Gilman, J., Jr., "Experimental Investigation of the Transonic Flutter of Simple Thin Truncated-Cone Panels," Langley Research Center, NASA TM X-599, October 1961.

60. Giltner, T. A. and Young, J. P., "Panel Flutter Test Report," Eng. Rep. No. 10, 235-M, The Martin Company, April 1958.

61. Giltner, T. A. and Young, J. P., "Flutter Characteristics of Stage I Between-Tanks Panels on the Titan Lot B Missile," Martin Company, Rpt. WDD-M-SR-58-46, August 1958.

62. Goland, M. and Luke, Y. L., "An Exact Solution for Two-Dimensional Linear Panel Flutter at Supersonic Speeds," Journal of Aeronautical Sciences, 21, 4, pp. 275-276, April 1954.

63. Golden, C. T., Hager, T. R., and Sherman, L. L., "Orthotropic Panel Flutter Analysis Correlation," D2-81301, Boeing Company, 1964.

64. Goldenveiser, A. L., "Teoria uprugikh tonkikh obolochek (Theory of Elastic Thin Shells)," Gosizdat Tekhnikoteoretich literatury, Moscow, 1953.

65. Goodman, L. E. and Rattayya, J. V., "Review of Panel Flutter and Effects of Aerodynamic Noise, I - Panel Flutter," Applied Mechanics Reviews, pp. 2-7, January 1960.

66. Greenspon, J., "Flutter of Thin Panels at Subsonic and Supersonic Speeds," Martin Company Rpt. ER 9934, November 1957.

67. Grigolyuk, E. I., "Small Vibrations of Thin Elastic Conical Shells," Izvestia, A. N. USSR Otdelnye Tekhnicheski Nauk, No. 6, pp. 35-44, 1956.

68. Guy, L. D., "The Effects of Aerodynamic Heating on Panel Flutter," Symposium Proceedings - Structural Dynamics of High Speed Flight, pp. 123-139, April 1961.

69. Guy, L. D. and Bohon, H. L., "Flutter of Aerodynamically Heated Aluminum-Alloy and Stainless-Steel Panels with Length-Width Ratio of 10 a Mach Number of 3.0," NASA TN D-1353, July 1962.

70. Guy, L. D. and Dixon, S. C., "A Critical Review of Experiment and Theory for Flutter of Aerodynamically Heated Panels," USAF (OSR) Symposium 62J31-4, October 1962.

71. Guy, L. D., "The Effects of Aerodynamic Heating on Panel Flutter," Symposium Proceedings-Structural Dynamics of High Speed Flight, pp. 123-139, April 1961.

72. Hains, F. D., "Flutter of a Thin Membrane in Hypersonic Flow," Journal of Aeronautical Sciences, 25, 9, p. 595, September 1958.

73. Hanson, P. W. and Levey, G. M., "Experimental and Calculated Results of a Flutter Investigation of Some Very Low Aspect Ratio Flat-Plate Surfaces at Mach Numbers from 0.62 to 3.00," NASA TM X-53, August 1959.

74. Hanson, P. W., "Experimental and Calculated Results of a Flutter Investigation of Some Very Low Aspect-Ratio and Flat-Plate Surfaces at Mach Numbers from 0.62 to 3.00," NASA TN D-2038, December 1963.

75. Hayes, W. D., "A Buckled Plate in a Supersonic Stream," North American Aviation Report AL-1029, May 1950.

76. Hedgepeth, J. M., Budiansky, B. and Leonard, R. W., "Analysis of Flutter in Compressible Flow of a Panel on Many Supports," Journal of Aeronautical Sciences, 21, 7, pp. 4750486, July 1954.

77. Hedgepeth, J. M., "On the Flutter of Panels at High Mach Numbers," Journal of Aeronautical Sciences, 23, 6, June 1956.

78. Hedgepeth, J. M., "Flutter of Rectangular Simply Supported Panels at High Supersonic Speeds," Journal of Aeronautical Sciences, 24, 8, pp. 563-573, August 1957.

79. Herrmann, G. and Mirsky, I., "On Vibrations of Conical Shells," Journal of Aeronautical Sciences, 25, 7, pp. 451-458, 1958.

80. Hess, R. W. and Gibson, F. W., "Experimental Investigation of the Effects of a Curved Panel and a Flat Panel at Supersonic Mach Numbers," NASA TN D-1386, October 1962.

81. Hess, R. W., and Osborn, J. A., "Some Effects of Differential Pressure and Damping Compound on the Flutter of Flat Panels Similar to Panels on the S-IV B Stage," NASA Langley Research Center, November, 1965.

82. Hess, R. W., "Preliminary Results of an Experimental Study of the Effects of Compressive Stress on Panel Flutter" NASA Langley Research Center, Jan. 1966.

83. Hodson, C. H. and Stocker, J. E., "Commercial Supersonic Transport Panel Flutter Studies," RTD-TDR-63-4036, May 1964.

84. Holt, M., "Aerodynamic Forces on a Cylindrical Shell in Panel Flutter," AFOSR TN 58-974, Brown University, 1958.

85. Holt, M. and Strack, S. L., "Supersonic Panel Flutter of a Cylindrical Shell of Finite Length," Journal of Aerospace Sciences, 28, 3, page 197, March 1961.

86. Houbolt, J. C., "A Study of Several Aerothermoelastic Problems of Aircraft Structures," Doctoral Thesis, E. T. H., Zurich, 1958.

87. Isaacs, R. P., "Transtability Flutter of Supersonic Aircraft Panels," Rand Report P-101, Rand Corporation, July 1, 1949.

88. Ishii, Takao, "Aeroelastic Instabilities of Simply Supported Panels in Subsonic Flow," AIAA Paper 65-772, November 15-18, 1965.

89. Johns, D. J., "Some Panel-Flutter Studies Using Piston Theory," Journal of Aeronautical Sciences, 25, 11, pp. 679-684, November 1958.

90. Johns, D. J., "The Influence of Panel Deformations on Wing Flutter," Reader's Forum, Journal of Aerospace Sciences, 27, 2, p. 137, February 1960.

91. Johns, D. J. and Parks, P. C., "The Effect of Structural Damping on Panel Flutter," Aircraft Engineering Report No. 32:304, October 1960.

92. Johns, D. J., Davies, R. J. and Nayler, G. H. F., "The Flutter of Thin Plane, Elliptical Panels," British-COA Note-136, January 1963.

93. Johns, D. J., "Some Panel Aeroelastic Instabilities," N.A.T.O. (AGARD) 474, September 1963.

94. Johns, D. J., "A Review of Panel Flutter at Low Supersonic Speeds," N.P.L. Aero Report 1102, April 1964.

95. Johns, D. J., "A Survey of Panel Flutter," Working Paper F3, AGARD, October 1964.

96. Johns, D. J., "The Present Status of Panel Flutter," N.A.T.O. (AGARD) 484, October 1964.

97. Johns, D. J., "A Survey on Panel Flutter," November 1965.

98. Johnson, M. W., "On the Dynamics of Shallow Elastic Membranes," University of Wisconsin MRC Technical Summary Report No. 116, December 1959.

99. Jones, W. P., General Editor, "Manual on Aeroelasticity," National Physical Laboratory England, NATO AGARD MANUAL 61A01, Vol. 3, Pt. 3.

100. Jordan, P., Greenspon, E. and Goldman, R., "Flutter of Thin Panels at Subsonic and Supersonic Speeds," AFOSR TR 57-65, Martin Aircraft, Baltimore, 1957.

101. Jordan, P., "The Physical Nature of Panel Flutter," Aero Digest, pp. 34-38, February 1956.

102. Jordan, P. F , "Remarks on Panel Flutter," Martin Company Report ER-7914.

103. Kelley, R. E., "Stability of Panels in Incompressible Unsteady Flows," AIAA Journal, June 1965.

104. Ketter, D. J. and Voss, H. M., "Panel Flutter Analyses and Experiments in the Mach Number Range of 5.0 to 10.0," Report No. ASD-TDR 64-6, November 1963.

105. Kobett, D. R. and Zeydel, E. F. E., "Research on Panel Flutter," NASA TN D-2227, November 1963.

106. Kobett, D. R., "Research on Panel Flutter," NASA CR-80, July 1964.

107. Kobet, D. R., "Flutter of Multi-Streamwise Bay Rectangular Panels at Low Supersonic Mach Number," NASA CR-538, August 1966.

108. Kopzon, G. I., "Vibration of Thin-Walled Elastic Bodies in a Gas Stream," Dokladi Akad. Nauk USSR (N.S.), 107, 2, pp. 217-220 (in Russian), 1956. Applied Mechanics Reviews, 10, Rev. 33, 1957.

109. Kopzon, G. I., "Vibrations of a Shallow Wing Shell in a Gas Flow," Dokladi Akad. Nauk USSR (N.S.), 107, 3, pp. 377-380 (in Russian), 1956. Applied Mechanics Reviews, 10, Rev. 34, 1957.

110. Kordes, E. E. and Evans, E. W., "Flutter Tests of Sandwich-Type Flat Panels," NASA Memo 10-17-58L, December 1958.

111. Kordes, E. E. and Noll, R. B., "Flight Flutter Results for Flat Rectangular Panels," NASA TN D-1058, February 1962.

112. Kordes, E. E. and Noll, R. B., "Theoretical Flutter Analyses of Flat Rectangular Panels in Uniform Coplanar Flow with Arbitrary Direction," NASA TN D-1156, January 1962.

113. Kordes, E. E., Tuovila, W. J. and Guy, L. D., "Flutter Research on Skin Panels," NASA Technical Note D-451, September 1960.

114. Krumhaar, Hans, "Supersonic Flutter of a Cylindrical Shell of Finite Length in an Axisymmetrical Mode," AFOSR 1574, October 1961.

115. Krumhaar, Hans, "The Accuracy of Applying Linear Piston Theory to Cylindrical Shells," CIT(GAL) SM-62-50, March 1963. (Also AIAA Journal Vol. 1, No. 6, June 1963.)

116. Lancaster, P., "Direct Solution of the Flutter Problem," British Aeronautical Research Council, R. and M. No. 3206.

117. Langley, B. L., Capt., and Bozich, W. R., Capt. USAF, "A Study of Panel Thickness Required to Prevent Flutter in Advanced Performance Vehicles," AFFDL-TR-65-164, February 1966.

118. Lemley, C. E., "The Effect of Inplane Edge Restraint on the Vibration and Flutter of Slightly Curved Panels," D.Sc. Thesis, Department of Applied Mechanics, Washington University, St. Louis, January 1967.

119. Lentz, R. G. and Spiroff, C. M. "ASSET Vol. IX, Flutter Panel Development AFFDL-TR-65-31, August 1965.

120. Leonard, R. W. and Hedgepeth, J. M., "On Panel Flutter and Divergence of Infinitely Long Unstiffened and Ring-Stiffened Thin-Walled Circular Cylinders," NACA TN 3638, April 1956. NACA Report 1302, 1957.

121. Leonard, R. W. and Hedgepeth, J. M., "On the Flutter of Infinitely Long Panels on Many Supports," Journal of Aeronautical Sciences, 24, 5, pp. 381-383, May 1957.

122. Leonard, R. W. and Hedgepeth, J. M., "Status of Flutter of Flat and Curved Panels," NACA RM L57D24C, May 28, 1957.

123. Lock, M. H. and Farkas, E. F., "The Flutter of Two-Bay Flat Panels of Infinite Span at Supersonic Mach Numbers," Report No. SSD-TDR-64-164, August 1964.

124. Lock, M. H., "Supersonic Flutter of Two-Dimensional Flat Panels - An Experimental Evaluation of the Theory," Proceedings of Fourth U.S. National Congress of Applied Mechanics, Vol. 2, 1962.

125. Lock, M. H., "Flutter of Two-Dimensional Panels in Transonic Regime," First Award Papers of Institute of Aeronautical Sciences, 1958. Progress Report to AFOSR, 1958.

126. Lock, M. H. and Fung, Y. C., "Comparative Experimental and Theoretical Studies of the Flutter of Flat Panels in a Low Supersonic Flow," AFOSR TN 670, California Institute of Technology, May 1961.

127. Luke, Y. L. and St. John, A., "Supersonic Panel Flutter," WADC TR 57-252, Midwest Research Institute, 1957.

128. Luke, Y. L., St. John, A. and Goland, M., "An Exact Solution for Two-Dimensional Linear Panel Flutter at Supersonic Speeds," WADC TN 56-460, Midwest Research Institute, March 15, 1956.

129. Luke, Y. L., Goland, M. and Constant, P., "Panel Flutter at Supersonic Speeds," Progress Report, AF33(616)-2897, Midwest Research Institute, June 1955.

130. Luke, Y. L., St. John, A. D. and Clark, Carol, "An Exact Solution for Two-Dimensional Linear Panel Flutter, Second Quarterly Progress Report, AF33(616)-2897, Midwest Research Institute, September 15, 1955.

131. Makhortykh, Z. K., "Stability of Multi-Bay Panels in High Supersonic Flow," Akad. Nauk USSR, Otd, Tekh, Nauk, Isvestia Mekh. i Mashinostr. (in Russian), pp. 174-177, March-April 1959.

132. McClure, J. D., "Stability of Finite Chord Panels Exposed to Low Supersonic Flows with a Turbulent Boundary Layer," IAS Paper 63-21, January 1963.

133. McElman, J. A., "Flutter of Curved and Flat Sandwich Panels Subjected to Supersonic Flow, NASA TN D-2192, April 1964.

134. Meekishef, G. N., "Experimental Investigation of the Flutter of a Square Plate in Supersonic Flow," Akad. Nauk USSR, Otd. Tekh. Nauk, Izvestia Mekh. i. Mashinostr. (in Russian), pp. 154-157, January-February 1959.

135. Mikulas, M. M. Jr., and McElman, J. A., "On Free Vibration of Eccentrically Stiffened Cylindrical Shells and Flat Plates," Langley Research Center, NASA-TN-D-3010, September 1965.

136. Miles, J. W., "The Aerodynamic Forces on an Oscillating Airfoil at Supersonic Speeds," Journal of Aeronautical Sciences, 14, 6, pp. 351-358, June 1947.

137. Miles, J. W., "Dynamic Chordwise Stability at Supersonic Speeds," North American Aviation Report AL-1140, October 1950.

138. Miles, J. W., "Supersonic Flutter of a Cylindrical Shell," The Ramo-Woolridge Corporation Guided Missile Res. Div.
I. General Theory Report No. AM5-2, August 19, 1955.
II. Pressurization and Internal Fluid Effects, Report No. AM5-11, November 14, 1955.
III. Aelotropic Shell, Report No. AM5-12, December 2, 1955.
IV. Effects of Non-Uniform Steady Flow, Report No. AM5-16, December 7, 1955.

139. Miles, J. W., "On the Aerodynamic Stability of Thin Panels," Journal of Aeronautical Sciences, 23, 8, pp. 771-780, August 1956.

140. Miles, J. W., "On a Reciprocity Condition for Supersonic Flutter," Journal of Aeronautical Sciences, 24, 12, p. 920, December 1957.

141. Miles, J. W., "Supersonic Panel Flutter of a Cylindrical Shell," I - Journal of Aeronautical Sciences, 24, 2, pp. 107-118, February 1957. II - Journal of Aeronautical Sciences, 25, 5, pp. 312-316, May 1958.

142. Miles, J. W., "On Panel Flutter in the Presence of a Boundary Layer," Journal of Aerospace Sciences, 26, 2, pp. 81-93, February 1959.

143. Miles, J. W., "On Supersonic Flutter of Long Panels," Journal of Aerospace Sciences, 27, 6, p. 476, June 1960.

144. Miles, J. W. and Rodden, W. P., "On the Supersonic Flutter of Two-Dimensional Infinite Panels," Journal of Aerospace Sciences, 26, 3, p. 190, March 1959.

145. Min-De, Dun, "On the Stability of Elastic Plates in a Supersonic Stream," Soviet Physics, Doklady (translated from Russian), pp. 479-483, May-June 1958.

146. Mirowitz, L. I., Zimmerman, N. H. and Schweiker, J. W., "Panel Flutter Survey and Design Criteria," ATC Report No. ARTC-32, August 1962.

147. Mitchell, D. H., "Flutter of a Buckled Plate," Massachusetts Institute of Technology, S. M. Thesis, 1957.

148. Movchan, A. A., "On Vibrations of a Plate Moving in a Gas," Prikladnaia Matematika i Mekhanika, 20, 2, 1956. (Recently translated and issued as NASA RE 11-22-58W.)

149. Movchan, A. A., "On the Stability of Panel Moving in a Gas," Prikladnaia Matematika i Mekhanika, 21, 2, 1957. (Recently translated and issued as NASA RE 11-21-58W.)

150. Mortvedt, R. L., Rich, R. L., and Wagner, R. T., "Skin Panel Flutter Test, Refine I," AD-375D-4, Boeing Co, 1962.

151. Mortvedt, R. L. and Wagner, R. T., "Transonic, Supersonic Panel Flutter Tests," Boeing Co. Report D2-81095, December 1963.

152. Muhlstein, L., "A Forced Vibration Technique for Investigation of Panel Flutter," AIAA Paper No. 66-769.

153. Nash, W. A., "Response of an Elastic Plate to a Distributed Random Pressure Characterized by a Separable Cross Correlation," AFOSR-TN-61-1215, September 1961.

154. Nelson, H. C. and Cunningham, H. J., "Theoretical Investigation of Flutter of Two-Dimensional Flat Panels with One Surface Exposed to Supersonic Potential Flow," NACA TN 3465, July 1955. NACA Report 1280, 1956.

155. Nugent, J., "Airborne Panel Flutter Analysis," Martin Co. Report CR-62-122, April 1964.

156. Olsen, James J., "Local Aerodynamic Parameters For Supersonic and Hypersonic Flutter Analysis," AFFDL-TR-65-183, October 1965.

157. Parks, P. C., "A Stability Criterion for a Panel Flutter Problem Via the Second Method of Liapunov," Proceedings of the AIAA Symposium of Structural Dynamics and Aeroelasticity, Boston, Mass., August 1965.

158. Perkins, T. M., "Aeroelastic Stability of an Array of Full-Scale Panels from the Saturn SIVB Stage at Transonic Mach Numbers," AEDC-TR-67-9, February 1967.

159. Perlmutter, A. A., "On the Aeroelastic Stability of Orthotropic Panels in Supersonic Flow," Journal of Aeronautical Sciences, 29, No. 5, November 1962.

160. Pines, S., "An Elementary Explanation of the Flutter Mechanism," Proceedings of the National Specialists' Meeting on Dynamics and Aeroelasticity, Institute of Aeronautical Sciences, pp. 52-58, November 1958.

161. Postel, E. E., "Derivation of a Panel Flutter Criterion in Terms of Natural Mode Frequencies," Lockheed California Co. Report No. LR-17961, June 1964.

162. Postel, E. E., "Panel Flutter and Related Aeroelastic Investigations," USAF (FDL) (TDR 64-87), March 1965.

163. Postel, E. E., "Panel Flutter and Vibratory Response to Turbulence," USAF (FDL) (TR-65-68), October 1965.

164. Presnell, J. G., and McKinney, R. L., "Experimental Panel Flutter Results for Some Flat and Curved Titanium Skin Panels at Supersonic Speeds," NASA TN D-1600, January 1963.

165. Rattayya, J. V. and Goodman, L. E., "Bibliographical Review of Panel Flutter and Effects of Aerodynamic Noise," WADC TR 59-70, ASTIA Document No. 215448, University of Minnesota, June 1959.

166. Rattayya, J. V., "Supersonic Speed Flutter Analysis of Circular Panels with Edges Elastically Restrained Against Rotation," PhD. Thesis, University of Minnesota, (WADD TR 60-309), August 1960.

167. Reed, Robert E., Jr., "Comparison of Methods in Calculating Frequency of Corner Supported Rectangular Plates," NASA TN-D-3030, September, 1965.

168. Rich, R. L., "Preliminary Skin Panel Flutter Tests," Boeing Co. Report No. D2-8148, 1962.

169. Rodden, W. P., "Flutter of Multi-Bay Panels," AIAA Journal, August 1964.

170. Rodden, W. P., "The Flutter of Two-Dimensional Flat Panels with Equally Spaced Supports in a Supersonic Flow," University of California, Los Angeles, Ph.D. Thesis, October 1, 1957.

171. Rodden, W. P., "On Vibration and Flutter Analysis with Free-Free Boundary Conditions," Journal of Aerospace Sciences, 28, 1, p. 65, January 1961.

172. Schaeffer, H. G. and Heard, W. L., "Flutter of Panels Subjected to Non-Linear Temperature Distribution," AIAA Journal, October 1965.

173. Schaeffer, Harry G., "Supersonic Flutter of a Thermally Stressed Flat Panel With Uniform Edge Loads," NASA TN-D-3077, October, 1965.

174. Shen, S. F., "Non-Stationary Aerodynamics of a Two-Dimensional Bump in a Uniform Stream and Its Effect on the Vibration Characteristics of an Elastic Panel," MIT Tech. Report. (Contract No. N5 ori-07833, NR 0640259 May 1952).

175. Shen, S. F., "Flutter of a Two-Dimensional Simply-Supported Uniform Panel in a Supersonic Stream," Massachusetts Institute of Technology, Aeroelasticity and Structures Laboratory, Report No. 25.10, 1952.

176. Shen, S. F., "Supersonic Flutter of a Two-Dimensional Panel," Presented at the Supersonic Conference, University of Maryland, College Park, May 19-20, 1954.

177. Shen, S. F., "Remarks on 'An Exact Solution for Two-Dimensional Linear Panel Flutter at Supersonic Speeds'," Journal of Aeronuatical Sciences, 22, 9, September 1955.

178. Shen, S. F., "An Approximate Analysis of Nonlinear Flutter Problems," Journal of Aerospace Sciences, 26, 1, pp. 25-32, January 1959.

179. Shideler, J. L., Dixon, S. C., and Shore, C. P., "Flutter at Mach 3 of Thermally Stressed Panels and Comparison with Theory for Panels with Edge Rotational Restraint," Langley Research Center, NASA-TN-D-3498, August 1966.

180. Shirk, M. H. and Olsen, J. J., "Recent Panel Flutter Research and Applications," NATO, AGARD Report 475, September 1963.

181. Shulman, Y., "Vibration and Flutter of Cylindrical and Conical Shells," AFOSR TR 59-776, Massachusetts Institute of Technology, June 1959.

182. Smith, G. E., "Flutter of a Two-Dimensional Simply-Supported Buckled Panel with Elastic Restraint Against Edge Displacement," Royal Aircraft Establishment, Report Structures 269, April 1961.

183. Stearman, R. O., "Research on Panel Flutter of Cylindrical Shells," USAF (OSR) 64-0074, January 1964.

184. Stearman, R. O., "Flutter of a Ring of Panels," CIT(GAL) SM 62-35, July 1962.

185. Stearman, R. O., Lock, M. H. and Fung, Y. C., "Ames Tests on the Flutter of Cylindrical Shells," CIT(GAL) SM 62-37, December 1962.

186. Stearman, R. O., "Small Aspect Ratio Membrane Flutter," AFOSR TR 59-45, California Institute of Technology, January 1959. First Award Papers, Institute of Aeronautical Sciences, 1959.

187. Stepanov, R. D., "On the Flutter of Cylindrical Shells and Panels Moving in a Flow of Gas," Prikladnaia Matematika i Mekhanika, 21, 5, 1957. (Translated and issued as NACA TM 1438, 1958)

188. Stocker, James E., "A Comprehensive Review of Theoretical and Experimental Panel Flutter Investigations, "N.A.A. Inc. (Columbus) Rpt. NA61H-444, September 1961.

189. Strack, S. L. and Holt, M., "Supersonic Panel Flutter of a Cylindrical Shell of Finite Length, " AFOSR TN 59-547, Brown University, May 1959.

190. Sylvester, M. A., "Experimental Studies of Flutter of Buckled Rectangular Panels at Mach Numbers from 1.2 to 3.0 Including Effects of Pressure Differential and of Panel Width-Length Ratio, "NASA TN D-833, May 1961.

191. Sylvester, M. A., Nelson, H. C. and Cunningham, H. J., "Experimental and Theoretical Studies of Panel Flutter at Mach Numbers 1.2 to 3.0,"NACA RM L55E18B, 1955.

192. Sylvester, M. A. and Baker, J. E., "Some Experimental Studies of Panel Flutter at Mach Number 1.3, "NACA TN 3914, 1957.

193. Tuovila, W. J. and Presnell, J. G., "Supersonic Panel Flutter Test Results for Flat Fiberglass Sandwich Panels with Foamed Cores," NASA TN D-827, June 1961

194. Tuovila, W. J. and Hess, R. W., "Experimental Investigation of Flutter of Buckled Curved Panels Having Longitudinal Stringers at Transonic and Supersonic Speeds," NASA Memo 5-18-59L, June 1959.

195. Ungar, E. E., "Fundamentals of Statistical Energy Analysis of Vibrating Systems, AFFDL-TR-66-52, May 1966

196. Vlasov, V. S., "Basic Differential Equations in General Theory of Elastic Shells, "NACA TM 1241, 1951.

197. Voss, H. M., "The Effect of Some Practical Complications on the Flutter of Rectangular Panels,"USN (ONR) ACR-62, Vol. I, Symposium Proceedings, Structrual Dynamics of High Speed Flight, United States Navy, Office of Naval Research, April 1961.

198. Voss, H. M., "The Effect of an External Supersonic Flow on the Vibration Characteristics of Thin Cylindrical Shells,"Preprint No. 60-45, Institute of Aeronautical Sciences, Presented at the 28th Annual IAS Meeting, New York, January 27, 1960.

199. Warburton, G. B., "The Vibration of Rectangular Plates, "Proceedings of the Institution of Mechanical Engineers, Vol. 168, pp. 371-384, 1954.

200. Weeks, George E. and Shideler, John L., "Effect of Edge Loadings on the Vibration of Rectangular Plates with Various Boundary Conditions," NASA TN D-2815, May 1965.

201. Weidman, Deene J., "Experimental Flutter Results for Corrugation - Stiffened and Unstiffened Panels," March 1966.

202. White, R.P., "Hypersonic Flutter Model Results and Comparisons with Piston Theory Predictions, " ASD-TR-61-347.

203. Wielandt, H., "The Method of Iteration for Non-Self-Adjoint Linear Eigenvalue Problems," Aerodynamische Versuchsanstalt Goettingen, 1943. (Translation published by the Ministry of Aircraft Production, Great Britain, as MAP-VG-155)

204. Yates, J. E. and Zeydel, E. F. E., "Flutter of Curved Panels," AFOSR TR 59-136, Midwest Research Institute, September 1959.

205. Zeydel, E. F. E., "Large Deflection Panel Flutter,"AFOSR TN 1952, Midwest Research Institute, January 1962.

206. Zeydel, E. F. E., and Kobett,, D. R., "Effects of External Force Excitation on Panel Flutter, "Midwest Research Institute TR-389, December 1962.

207. Zeydel, E. F. E. and Horbelt, D. R., "Flutter of Flat Plates with Partially Clamped Edges in the Low Supersonic Region," IAS Paper 63-25, January 1963.

208. Zimmerman, N. H., "Panel Flutter Design Criteria, "McDonnell Aircraft Corporation Memo 525, 13 July 1959.

209. Zisfein, M. B. and Frueh, F. J., "A Study of Velocity-Frequency-Damping Relationships for Wing and Panel Binary Systems in High Supersonic Flow, "AFOSR TN 59-969, October 1959.

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13. ABSTRACT

The program described in this report was performed to bring together all available data from wind tunnel test, flight test, vibration test, thermal test and theoretical investigations to form comprehensive panel flutter design criteria. Procedures were developed which are applicable to the environment and various panel structural arrangements for transonic and hypersonic aircraft, aerospace reentry vehicles, and boosters.

This report (Volume II) presents the results of investigations to determine the state-of-the-art in panel design and to provide the background data for the criteria that are given in Volume 1. The investigations included a thorough literature search and review as well as surveys of personnel and facilities having made recent contributions in the field. In addition, supplementary analyses are described that were required in some areas to complete the criteria presentation. A comprehensive bibliography is appended to this volume.

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